Day 270 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:

$y = x^2 - 4x + 12$, $y = 2x + 4$.

$x^2 - 4x + 12 = 2x + 4$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

Use $y = 2x + 4$ to find each y.

Answer: x = 2, y = 8 and x = 4, y = 12

2 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + y^2 = 169$.

Find dy/dx in terms of x and y, and hence the gradient at $(5, -12)$.

Differentiate both sides with respect to x (using dy/dx for y terms)

$\frac{dy}{dx} = -\frac{x}{y}$

At $(5, -12)$: $\frac{dy}{dx} = \frac{5}{12}$

Answer: $\frac{dy}{dx} = -\frac{x}{y}$; gradient $\frac{5}{12}$

3 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 40 kg sits 2 m from the pivot on one side. Where must a child of mass 32 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:

$40\text{ g} \times 2 = 32\text{ g} \times d$

$d = \frac{40 \times 2}{32}$

Answer: 2.5 m from the pivot

4 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ to estimate sin 5°, giving your answer to 3 significant figures.

Convert to radians: $5° = \frac{5\pi}{180} = 0.08727$ rad

$\sin 5° \approx 0.0873$

Answer: 0.0873

5 Regression: interpreting coefficients (AQA, Edexcel, OCR, MEI, Eduqas)

A model relates the temperature, T °C, of a cooling drink to the time, t minutes, since pouring by $T = 88 - 1t$.

Interpret the value 88 in this context.

The constant is the predicted value of T when t = 0.

So the predicted T at t = 0 is 88 °C.

Answer: predicted T is 88 °C when t = 0

6 Parametric equations to Cartesian form (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 3$, $y = 4t^2 + 6$.

Find a Cartesian equation of the curve.

From $x = t + 3$: $t = x - 3$

Substitute into y: $y = 4(x - 3)^2 + 6$

Answer: $y = 4(x - 3)^2 + 6$

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7 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 30 kg and length 8 m rests horizontally on supports at A and B. A load of mass 10 kg is placed on the beam 4 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:

$R_B \times 8 = 30 \times 9.8 \times 4 + 10 \times 9.8 \times 4$

$R_B \times 8 = 1568$

Answer: 196 N

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve $y = x^2 - 6x + 4$.

Find the value of k.

$x^2 - 6x + 4 = -4x + k$

$x^2 - 2x + (4 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(4 - k) = 0$

Answer: k = 3

9 Double angle: find sin 2A and cos 2A (All boards)

A is acute and $\cos A = \frac{8}{17}$.

Find the exact values of sin 2A and cos 2A.

$\sin A = \sqrt{1 - \cos^2 A} = \frac{15}{17}$

$\sin 2A = 2 \sin A \cos A = 2 \times \frac{15}{17} \times \frac{8}{17} = \frac{240}{289}$

$\cos 2A = 2\cos^2 A - 1 = 2 \times \frac{64}{289} - 1 = -\frac{161}{289}$

Answer: sin 2A = $\frac{240}{289}$, cos 2A = $-\frac{161}{289}$

10 Box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plot summarises the journey times, in minutes, of some commuters.

Find the range and the interquartile range.

Range = 40 − 5 = 35

IQR = 35 − 25 = 10

Answer: range = 35, IQR = 10