Day 27 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$.

Write down a vector equation of the line through the origin, parallel to this line.

Parallel lines have the same direction vector

The line passes through O, so the position vector is the zero vector

Answer: $\mathbf{r} = s\begin{pmatrix} \mathbf{-1} \\ \mathbf{1} \\ \mathbf{3} \end{pmatrix}$

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = \frac{5}{2}x + 3$ is perpendicular to the line

$y = kx - 5$. Find the value of k.

Perpendicular lines have gradients with product -1.

$\frac{5}{2} \times k = -1$

Answer: $\mathbf{k = -\frac{2}{5}}$

3 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with temperature (°C) from 10 to 18, the regression line is S = 20 + 2T.

Estimate ice cream sales when T = 8 and comment on the reliability.

$S = 20 + 2 \times 8 = 36$

8 is outside the range 10 to 18, so this is extrapolation.

Answer: S = 36; estimate is unreliable, as 8 is outside the data range (extrapolation)

4 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.

Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length $= r\theta = 6 \times \frac{\pi}{3} = 2\pi$ cm

(b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 6^2 \times \frac{\pi}{3} = 6\pi$ cm^2

Answer: (a) 2π cm (b) 6π cm^2

5 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 7 terms of the geometric series 2 + 4 + 8 + ...

a = 2, r = 2, n = 7

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{2(2^7 - 1)}{2 - 1}$

$= 254$

Answer: 254

6 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation xy = 6.

Find dy/dx in terms of x and y, and hence the gradient at (2, 3).

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{y}{x}$

At (2, 3): $dy/dx = -\frac{3}{2}$

Answer: $\mathbf{dy/dx = -\frac{y}{x}}$**; gradient** $\mathbf{-\frac{3}{2}}$

7 Particle on a smooth table connected to a hanging particle (All boards)

A 8 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 6 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: T = 8 a
Hanging particle: 6 × 9.8 − T = 6 a
Adding: 58.8 = 14 a
Answer: 33.6 N

8 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−4, 4) and B is (−8, −2).
Find the equation of the perpendicular bisector of AB, in the form ax + by = c with integer a, b and c.

Midpoint of AB = (−6, 1)
Gradient of AB = $\frac{-6}{-4} = \frac{3}{2}$, so perpendicular gradient = $-\frac{2}{3}$
$y - 1 = -\frac{2}{3}(x - (-6))$
Multiply out and rearrange.
Answer: 2x + 3y = −9

9 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train travelling at 12 m s^{-1} brakes uniformly and comes to rest after travelling 48 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 12^2 + 2 \times a \times 48$
so a = −1.5, a deceleration of 1.5
(b) $s = \frac{(u + v)}{2}$ t: 48 = 6 t
Answer: (a) 1.5 m s^{-2} (b) 8 s

10 Normal approximation to the binomial (CIE)

X ~ B(200, 0.3). Use a normal approximation to find P(X ≤ 63), to 3 s.f.

np = 60 and npq = 42, so use Y ~ N(60, 42)
Continuity correction: P(Y < 63.5)
$z = (63.5 - 60) \div \sqrt{42}$
Answer: 0.705