Day 268 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Amplitude and period from a graph (diagram) (All boards)

The diagram shows the graph of $y = a \sin bx$ for $0° \leq x \leq 360°$, where a and b are positive integers.
Find a and b.

The maximum value of y is 4, so a = 4

The graph completes 3 full cycles in 360°, so the period is 120° and $b = \frac{360}{120} = 3$

Answer: a = 4, b = 3

2 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(6\mathbf{i})$ m s^{-1} and constant acceleration $(3\mathbf{i} + 2\mathbf{j})$ m s^{-2}.
Find its velocity after 3 s.

$v = u + at$
$v = (6\mathbf{i}) + 3(3\mathbf{i} + 2\mathbf{j})$
$= (6\mathbf{i}) + (9\mathbf{i} + 6\mathbf{j})$
Answer: $(15\mathbf{i} + 6\mathbf{j})$ m s^{-1}

3 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

10 pairs of data on hours of revision and test score have $r = -0.4$.
Test, at the 5% significance level, for evidence of negative correlation. The critical value is -0.5494.

$H_0: \rho = 0$, $H_1: \rho < 0$
$r = -0.4$ is not beyond the critical value -0.5494
Do not reject H_0
Answer: Do not reject H_0: there is not enough evidence of negative correlation

4 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - x - 9$, $y = -x - 5$.

$x^2 - x - 9 = -x - 5$
$x^2 - 4 = 0$
$(x + 2)(x - 2) = 0$
Use $y = -x - 5$ to find each y.
Answer: x = −2, y = −3 and x = 2, y = −7

5 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(8\mathbf{i} + 39.2\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the greatest height reached.

(a) Vertically: $0 = 39.2\,t - 4.9t^2$
$t = \frac{39.2}{4.9} = 8$
(b) $0 = 39.2^2 - 2 \times 9.8 \times H$
$H = \frac{1536.64}{19.6}$
Answer: (a) 8 s (b) 78.4 m

6 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 5^2)$. $H_0: \mu = 110$, $H_1: \mu > 110$.
A sample of 9 has mean $\bar{x} = 113.67$.
Test at the 5% level. Critical value: z = 1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (113.67 - 110) \div (5 \div 3)$
$z = 2.20$
2.20 is inside the critical region, so reject H_0
Answer: z = 2.20; reject H_0

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7 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -x + k$ is a tangent to the curve
$y = x^2 + 5x - 4$.
Find the value of k.

$x^2 + 5x - 4 = -x + k$
$x^2 + 6x + (-4 - k) = 0$
A tangent touches once: discriminant = 0
$(6)^2 - 4(-4 - k) = 0$
Answer: $k = -13$

8 Integrate using partial fractions (All boards)

$\frac{-x - 5}{(x + 3)(x + 1)} \equiv \frac{A}{x + 3} + \frac{B}{x + 1}$.
Find A and B, and hence find $\int \frac{-x - 5}{(x + 3)(x + 1)}\, dx$.

$-x - 5 \equiv A(x + 1) + B(x + 3)$
$x = -3$ gives $A = 1$; $x = -1$ gives $B = -2$
$\int \left(\frac{1}{x + 3} + \frac{-2}{x + 1}\right) dx = 1 \ln|x + 3| + -2 \ln|x + 1| + c$
Answer: $A = 1, B = -2$; $\ln|x + 3| - 2 \ln|x + 1| + c$

9 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is $(-4, 1)$ and B is $(6, 7)$. AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = (1, 4)
$r^2 = (6 - 1)^2 + (7 - 4)^2 = 34$
Equation: $(x - a)^2 + (y - b)^2 = r^2$
Answer: $(x - 1)^2 + (y - 4)^2 = 34$

10 Solve a logarithmic equation (one log) (All boards)

Solve $\log_x 16 = 4$.

$x^4 = 16$
$x = 2$
Answer: $x = 2$