Day 267 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve
$y = x^2 - 9x$.
Find the value of k.

$x^2 - 9x = -3x + k$
$x^2 - 6x + (0 - k) = 0$
A tangent touches once: discriminant = 0
$(-6)^2 - 4(0 - k) = 0$
Answer: $k = -9$

2 Constant acceleration: s = ut + ½at² (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moves in a straight line with constant acceleration 1.5 m s^{-2}. Its initial speed is 6 m s^{-1}. Find the distance travelled in 3 s.

$s = ut + \frac{1}{2}at^2$

$s = 6 \times 3 + \frac{1}{2} \times 1.5 \times 3^2$

Answer: 24.75 m

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{9x^4} \times 3x^4$.

$\sqrt{9x^4} = (9x^4)^{\frac{1}{2}} = 3\,x^2$
$3\,x^2 \times 3x^4 = 9x^6$
Answer: $9x^6$

4 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 5t + 4)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 5t + 4 = 0$
$(t - 1)(t - 4) = 0$
(b) $a = \frac{dv}{dt} = 2t - 5$; at $t = 1$, $a = 2 - 5$
Answer: (a) t = 1 and t = 4 (b) -3 m s^{-2}

5 Addition formulae: exact values (All boards)

By writing 75° as 45° + 30°, find the exact value of cos 75°.

$\cos(45° + 30°) = \cos 45° \cos 30° - \sin 45° \sin 30°$
$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$
Simplify: $\frac{\sqrt{6} - \sqrt{2}}{4}$
Answer: $\frac{\sqrt{6} - \sqrt{2}}{4}$

6 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the height of a student chosen at random from a school.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here the outcome is a measurement, not success or failure in a fixed number of trials.
Answer: No, the outcome is a measurement, not success or failure in a fixed number of trials

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7 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + xy = 3$.
Find dy/dx in terms of x and y, and hence the gradient at (1, 2).

Differentiate both sides with respect to x (using dy/dx for y terms)

$\mathrm{dy/dx} = -\frac{2x + y}{x}$

At (1, 2): $\mathrm{dy/dx} = -4$

Answer: $\mathrm{dy/dx} = -\frac{2x + y}{x}$; gradient -4

8 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.5. If so, the probability that an item is faulty is 0.5; otherwise it is 0.2.
Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.5 × 0.5 = 0.25; P(not first and second) = 0.5 × 0.2 = 0.1
P(second) = 0.25 + 0.1 = 0.35
P = 0.25 ÷ 0.35

Answer: $\frac{5}{7}$ = 0.714

9 Find the angle of a sector (All boards)

A sector has radius 5 cm and area 21.9 cm^2.
Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 5^2 \times \theta = 21.9$, so $\theta = 1.75$

Arc length = $r\theta = 5 \times 1.75 = 8.75$ cm

Answer: θ = 1.75 radians; arc length 8.75 cm

10 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The nth term of an arithmetic sequence is 3n + 1.
Find the smallest n for which the term exceeds 200.

$3n + 1 > 200$

$n > \frac{199}{3} = 66.33...$

The smallest integer n is 67.

Answer: n = 67