Day 266 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = 2t + 2$, $y = t^2 + 4$.

The point P has y-coordinate 13 and positive t.

Find the coordinates of P.

$t^2 + 4 = 13$, so $t^2 = 9$, giving $t = 3$ ($t > 0$)

$x = 2 \times 3 + 2 = 8$

Answer: (8, 13)

2 Velocity-time graph: acceleration and distance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a train over 42 seconds as it moves in a straight line.

(a) Find its acceleration in the first 8 s.

(b) Find the total distance travelled.

(a) acceleration = gradient = $\frac{12}{8}$

(b) distance = area of trapezium

$= \frac{1}{2}(30 + 42) \times 12$

Answer: (a) 1.5 m s^{-2} (b) 432 m

3 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x = 3\cos t$, $y = 2\sin t$.

Find dy/dx in terms of t, and its value when $t = \frac{\pi}{4}$.

$dx/dt = -3\sin t$, $dy/dt = 2\cos t$

$dy/dx = \frac{2\cos t}{-3\sin t} = -\frac{2}{3}\cot t$

At $t = \frac{\pi}{4}$, $\cot t = 1$, so $dy/dx = -\frac{2}{3}$

Answer: $dy/dx = -\frac{2}{3}\cot t$; $-\frac{2}{3}$

4 Integrating velocity to find displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (3t^2 - 2t + 4)$ m s^{-1} at time t seconds.

Find the distance travelled between t = 1 and t = 4 (the particle does not turn round).

$s = \int v\,dt = t^3 - t^2 + 4t$

Distance $= s(4) - s(1)$

$= 64 - 4$

Answer: 60 m

5 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|4x - 1| = 9$.

$4x - 1 = 9$ or $4x - 1 = -9$

$x = \frac{5}{2}$ or $x = -2$

Answer: $x = \frac{5}{2}$ or $x = -2$

6 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 1 : 1.

Find AB and AP in terms of $\mathbf{a}$ and $\mathbf{b}$.

$AB = OB - OA = \mathbf{b} - \mathbf{a}$

$AP = \frac{1}{2}AB = \frac{1}{2}(\mathbf{b} - \mathbf{a})$

Answer: $AB = \mathbf{b} - \mathbf{a}$; $AP = \frac{1}{2}(\mathbf{b} - \mathbf{a})$

7 Counting: arrangements and selections (CIE)

How many different arrangements are there of the letters of the word LETTER?

6 letters, with repeats: E appears 2 times, T appears 2 times

$6! \div (2! \times 2!)$

Answer: 180

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 4x + k$ is a tangent to the curve $y = x^2 + 5$.

Find the value of k.

$x^2 + 5 = 4x + k$

$x^2 - 4x + (5 - k) = 0$

A tangent touches once: discriminant = 0

$(-4)^2 - 4(5 - k) = 0$

Answer: $k = 1$

9 Collinear points using vectors (All boards)

A(3, 2), B(15, 10) and C(30, k).

Given that A, B and C are collinear, find k.

$\text{AB} = \begin{pmatrix} 12 \\ 8 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} 27 \\ k - 2 \end{pmatrix}$

$\text{AC} = \frac{9}{4}\text{AB}$, so $27 = \frac{9}{4} \times 12$, giving the multiple $\frac{9}{4}$

$k - 2 = \frac{9}{4} \times 8$, so $k = 20$

Answer: $k = 20$

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 6^2)$. H_0: $\mu = 55$, H_1: $\mu \neq 55$.

A sample of 36 has mean $\bar{x} = 56.2$.

Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (56.2 - 55) \div (6 \div 6)$

$z = 1.20$

1.20 is outside the critical region, so do not reject H_0

Answer: $z = 1.20$; do not reject H_0