Day 265 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Normal distribution: symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(20, 10^2)$ and $P(X > 35) = 0.07$.
Find $P(5 < X < 35)$.

By symmetry $P(X < 5) = 0.07$
Centre $= 1 - 2 \times 0.07$
Answer: 0.86

2 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A train moving in a straight line increases its speed from 7 m s^{-1} to 21 m s^{-1} with constant acceleration 1 m s^{-2}. Find the time taken.

$v = u + at$
$21 = 7 + 1\,t$
Answer: 14 s

3 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) sec 240° (b) cosec 240°

(a) $\sec 240° = \dfrac{1}{\cos 240°} = \dfrac{1}{-\frac{1}{2}} = -2$

(b) $\operatorname{cosec} 240° = \dfrac{1}{\sin 240°} = \dfrac{1}{-\frac{\sqrt{3}}{2}} = -\dfrac{2\sqrt{3}}{3}$

Answer: (a) -2 (b) $-\dfrac{2\sqrt{3}}{3}$

4 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form $px + qy = r$ with integers p, q and r, (b) the length AB.

Gradient $= \dfrac{2 - 0}{0 - (-6)} = \dfrac{1}{3}$

Using $y = mx + c$ with $c = 2$: $y = \frac{1}{3}x + 2$, so $x - 3y = -6$

$AB^2 = 6^2 + 2^2 = 40$
Answer: (a) $x - 3y = -6$ (b) $2\sqrt{10}$

5 Product rule (All boards)

Use the product rule to differentiate
$y = x^2 \ln x$.

$u = x^2$, $v = \ln x$
$u' = 2x^1$, $v' = \dfrac{1}{x}$
$dy/dx = u'v + uv'$, which simplifies to the answer below
Answer: $dy/dx = x^1(2 \ln x + 1)$

6 Proof by exhaustion (AQA, Edexcel, OCR, MEI, Eduqas)

Use proof by exhaustion to show that $n^3 - n$ is divisible by 6 for every integer n with $1 \le n \le 5$.

Substitute each value of n:
n = 1: $0 = 6 \times 0$; n = 2: $6 = 6 \times 1$
n = 3: $24 = 6 \times 4$; n = 4: $60 = 6 \times 10$
n = 5: $120 = 6 \times 20$
Every value is a multiple of 6, so the statement is true for $1 \le n \le 5$.
Answer: Shown

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7 Maximum value from R form (All boards)

f(x) = 3 sin x + 4 cos x + 4.
Write f(x) in the form R sin(x + α) + 4, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{3^2 + 4^2} = 5$, $\tan \alpha = \frac{4}{3}$ so $\alpha = 53.1°$

f(x) = 5 sin(x + 53.1°) + 4
sin(x + α) has maximum value 1, so f(max) = 5 + 4
Answer: f(x) = 5 sin(x + 53.1°) + 4; maximum 9

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 4x + k is a tangent to the curve
$y = x^2 + 6x$.
Find the value of k.

$x^2 + 6x = 4x + k$
$x^2 + 2x + (0 - k) = 0$
A tangent touches once: discriminant = 0
$(2)^2 - 4(0 - k) = 0$
Answer: k = −1

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, p). H_0: p = 0.2, H_1: p > 0.2.
Find the critical region for a test at the 10% significance level.

P(X ≥ 9) = 0.0468 ≤ 0.1
P(X ≥ 8) = 0.109 > 0.1
Answer: X ≥ 9

10 Resultant of perpendicular forces (All boards)

Two forces of magnitudes 8 N and 15 N act on a bead at right angles to each other. Find (a) the magnitude of the resultant, (b) the angle it makes with the 8 N force, to 1 d.p.

(a) $R^2 = 8^2 + 15^2 = 289$
$R = \sqrt{289}$
(b) $\tan \theta = \frac{15}{8}$
Answer: (a) 17 N (b) 61.9°