Day 264 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) 4 sin 2x
(b) 4 ln x

(a) dy/dx = 8 cos 2x

(b) dy/dx = $\frac{4}{x}$

Answer: (a) 8 cos 2x (b) $\frac{4}{x}$

2 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In an arithmetic sequence $u_5 = 6$ and $u_{12} = 20$.
Find the first term and the common difference.

Subtract: $(12 - 5)d = 20 - 6$
$7d = 14$, so $d = 2$
$a = u_5 - 4d = 6 - 8 = -2$
Answer: $a = -2$, $d = 2$

3 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = -5\mathbf{i} + 12\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{5^2 + 12^2} = 13$

(b) Acute angle = $\tan^{-1}(\frac{12}{5}) = 67.4°$

p is in the second quadrant, so the angle is 112.6°
Answer: (a) 13 (b) 112.6°

4 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $x^2 + (y + 3)^2 = 169$ passes through P(5, 9).
Find the gradient of the tangent to the circle at P.

Centre C(0, −3).

Gradient of CP = $\frac{9 - (-3)}{5 - 0} = \frac{12}{5}$

The tangent is perpendicular to CP: gradient = $-1 \div \frac{12}{5}$

Answer: $-\frac{5}{12}$

5 Grouped data: estimated median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $0 \le x < 5$; $5 \le x < 10$; $10 \le x < 15$; $15 \le x < 20$
Frequency: 5, 6, 3, 9
Estimate the median, to 3 s.f.

n = 23, so the median is the 11.5 th value, in class $10 \le x < 15$
Cumulative frequency before this class = 11

Median $\approx 10 + \frac{(11.5 - 11)}{3} \times 5$

Answer: 10.8

6 Friction: acceleration along rough ground (All boards)

A block of mass 5 kg is pulled along rough horizontal ground by a horizontal force of 32.1 N.
The coefficient of friction is 0.4. Take g = 9.8 m s^{-2}. Find the acceleration.

$R = 5 \times 9.8 = 49$
$F = \mu R = 0.4 \times 49 = 19.6$
$P - F = ma$: $32.1 - 19.6 = 5a$
Answer: 2.5 m s^{-2}

7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = -2$ and $y = 2$, so $a = -2$ and $c = 2$.

Through (0, 1): $1 = \frac{k}{0 - (-2)} + 2$

$\frac{k}{2} = -1$, so $k = -2$

Answer: $a = -2$, $c = 2$, $k = -2$

8 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx^3$ for $0 \le x \le 1$, and 0 otherwise. Find the value of k.

$\int$ from 0 to 1 of $k\,x^3\,dx = 1$

$k \times \frac{1^4}{4} = 1$

Answer: $k = 4$

9 Free fall: dropped from a height (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A ball is dropped from rest from the top of a tower of height 44.1 m. Take $g = 9.8$ m s^{-2} and ignore air resistance. Find (a) the time taken to reach the ground, (b) its speed on hitting the ground.

(a) $s = ut + \frac{1}{2}at^2$: $44.1 = \frac{1}{2} \times 9.8 \times t^2$

$44.1 = 4.9t^2$, so $t^2 = 9$

(b) $v = u + at = 9.8 \times 3$

Answer: (a) 3 s (b) 29.4 m s^{-1}

10 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 9x^2 + 24x + 5$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 18x + 24$

$dy/dx = 0$ gives $x = 2$ or $x = 4$

Substitute: $y = 25$ at $x = 2$ and $y = 21$ at $x = 4$

Answer: A (2, 25), B (4, 21)