Day 263 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(−6, 0) is the midpoint of the line segment AB.
A is the point (−5, −5). Find the coordinates of B.

x-coordinate: $\frac{-5 + x_B}{2} = -6$, so $x_B = -7$

y-coordinate: $\frac{-5 + y_B}{2} = 0$, so $y_B = 5$

Answer: B(−7, 5)

2 Solve a^x = b using logarithms (All boards)

Solve $2 \times 7^x = 20$.
Give your answer to 3 significant figures.

Divide by 2: $7^x = 10$

$x = \frac{\log 10}{\log 7} = 1.18$

Answer: x = 1.18

3 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car travelling at 20 m s^{-1} brakes uniformly and comes to rest after travelling 40 m.
(a) Find the deceleration.
(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 20^2 + 2 \times a \times 40$
so $a = -5$, a deceleration of 5

(b) $s = \frac{(u + v)}{2} t$: $40 = 10\,t$

Answer: (a) 5 m s^{-2} (b) 4 s

4 Rearrange and iterate (All boards)

(a) Show that $x^3 - 2x - 3 = 0$ can be rearranged to $x = (2x + 3)^{\frac{1}{3}}$.
(b) Use $x_{n+1} =$ this formula with $x_0 = 2$ to find x_1, x_2 to 3 d.p.

(a) $x^3 = 2x + 3$, so $x = \sqrt[3]{(2x + 3)}$
(b) $x_1 = 1.9129$, $x_2 = 1.8969$

Answer: (a) shown (b) $x_1 = 1.913$, $x_2 = 1.897$

5 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 2x - 1 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 1)^2 - 1 - 1 = 0$

$(x + 1)^2 = 2$

$x + 1 = \pm\sqrt{2}$

Answer: $x = -1 \pm \sqrt{2}$

6 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-4\mathbf{i} - 4\mathbf{j})$ N, $(2\mathbf{i})$ N and $(p\mathbf{i} + q\mathbf{j})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:
i components: $p + (-4) + (2) = 0$
j components: $q + (-4) + (0) = 0$

Answer: p = 2, q = 4

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7 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of heads when a fair coin is tossed 8 times.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here a fixed number of independent trials, each with probability 0.5 of a head.
Answer: Yes, a fixed number of independent trials, each with probability 0.5 of a head

8 Partial fractions: find the constants (All boards)

Express $\frac{-4x-14}{(x+3)(x+5)}$ in the form $\frac{A}{x+3}+\frac{B}{x+5}$.
Find the values of A and B.

$-4x-14 \equiv A(x+5)+B(x+3)$
$x=-3$: $-2=A(2)$, so $A=-1$
$x=-5$: $6=B(-2)$, so $B=-3$
Answer: $A=-1$, $B=-3$

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a patient takes the drug is 0.6. If so, the probability that the patient recovers is 0.7; otherwise it is 0.3.
Given that the patient recovers, find the probability that a patient takes the drug.

P(both) = 0.6 × 0.7 = 0.42; P(not first and second) = 0.4 × 0.3 = 0.12
P(second) = 0.42 + 0.12 = 0.54
P = 0.42 ÷ 0.54
Answer: $\frac{7}{9}$ = 0.778

10 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The nth term of an arithmetic sequence is 3n + 2.
Find the smallest n for which the term exceeds 400.

$3n+2>400$
$n>\frac{398}{3}=132.67...$
The smallest integer n is 133.
Answer: n = 133