Day 262 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Transforming a graph with a labelled turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). The maximum point is shown. Write down the coordinates of the maximum point on the curve with equation
(a) y = f(x + 4) (b) y = 2f(x)

(a) y = f(x + 4): the maximum point moves to (−8, 5).
(b) y = 2f(x): the maximum point moves to (−4, 10).
Answer: (a) (−8, 5) (b) (−4, 10)

2 Arc length and perimeter of a sector (All boards)

A sector has radius 15 cm and angle $\frac{5\pi}{6}$ radians.

Find the arc length, leaving π in your answer, and the perimeter.

Arc length = $r\theta = 15 \times \frac{5\pi}{6} = \frac{25\pi}{2}$ cm

Perimeter = arc + 2r = $\frac{25\pi}{2} + 30$ cm

Answer: Arc $\frac{25\pi}{2}$ cm; perimeter $(\frac{25\pi}{2} + 30)$ cm

3 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 5x^2 - 4x + 20$
(a) Show that (x + 2) is a factor of f(x).
(b) Factorise f(x) completely.

f(−2) = (−8) + (−20) + 8 + 20 = 0, so (x + 2) is a factor.
$f(x) = (x + 2)(x^2 - 7x + 10)$
$x^2 - 7x + 10 = (x - 2)(x - 5)$
Answer: (b) f(x) = (x + 2)(x − 2)(x − 5)

4 Starting from rest: find the time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A skier starts from rest and moves in a straight line with constant acceleration 4 m s^{-2}.
It travels 98 m. Find the time taken.

$s = ut + \frac{1}{2}at^2$ with u = 0

$98 = \frac{1}{2} \times 4 \times t^2$

$t^2 = 49$
Answer: 7 s

5 Box plots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The box plot summarises the journey times, in minutes, of some commuters.
Find the range and the interquartile range.

Range = 60 − 20 = 40
IQR = 45 − 30 = 15
Answer: range = 40, IQR = 15

6 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 6 kg and length 6 m is supported by a pivot 0.5 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:
m g × 0.5 = 6 g × (3 − 0.5)
$m = \frac{6 \times 2.5}{0.5}$
Answer: 30 kg

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7 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 4 \ln x + x$ at the point where $x = 1$.

$dy/dx = 1 + 4/x$, so the gradient of the curve at $x = 1$ is 5
At $x = 1$, $y = 1$
$y - 1 = 5(x - 1)$
Answer: $y = 5x - 4$

8 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 4^2)$. H_0: $\mu = 45$, H_1: $\mu > 45$.

A sample of 9 has mean $\bar{x} = 48.87$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (48.87 - 45) \div (4 \div 3)$
$z = 2.90$
2.90 is inside the critical region, so reject H_0
Answer: $z = 2.90$; reject H_0

9 Chain rule: gradient at a point (All boards)

Find the gradient of the curve $y = \sqrt{2x + 3}$
at the point where $x = 3$.

Use the chain rule: $dy/dx = \dfrac{2}{2\sqrt{2x + 3}}$

At $x = 3$: $dy/dx = \dfrac{1}{3}$

Answer: $\dfrac{1}{3}$

10 Algebraic division (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Divide $f(x) = x^4 + 6x^3 + 4x^2 - 11x - 12$ by $(x + 2)$.
Find the quotient and the remainder.

Use synthetic division with $a = -2$:
Bottom row: 1, 4, −4, −3, −6
Quotient: $x^3 + 4x^2 - 4x - 3$
Remainder: −6
Answer: Quotient $x^3 + 4x^2 - 4x - 3$, remainder −6