Day 261 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a particle is $(8\mathbf{i} + 15\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.
(a) Find the speed.
(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{8^2 + 15^2} = \sqrt{289}$

(b) $\tan\theta = \frac{15}{8}$

$\theta = \tan^{-1}(\frac{15}{8})$

Answer: (a) 17 m s^{-1} (b) 61.9°

2 Discrete random variable: E(X) and Var(X) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has this distribution.
x: 5, 6, 7, 8
P(X = x): 0.3, 0.3, 0.2, 0.2
Find E(X) and Var(X).

$E(X) = \Sigma xp = 5\times0.3 + 6\times0.3 + 7\times0.2 + 8\times0.2 = 6.3$

$E(X^2) = \Sigma x^2 p = 40.9$

$\text{Var}(X) = 40.9 - 6.3^2$

Answer: E(X) = 6.3, Var(X) = 1.21

3 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−5, 4) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(x) + 1 (b) y = 3f(x)

(a) y = f(x) + 1 adds 1 to y, so P maps to (−5, 5).
(b) y = 3f(x) multiplies y by 3, so P maps to (−5, 12).
Answer: (a) (−5, 5) (b) (−5, 12)

4 Binomial: modelling conditions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X is the number of sixes when a fair die is rolled 19 times.
Can X be modelled by a binomial distribution? Give a reason.

A binomial model needs: a fixed number of trials, two outcomes, constant probability, independent trials.
Here there is a fixed number of independent trials, each with the same probability of a six.
Answer: Yes, there is a fixed number of independent trials, each with the same probability of a six

5 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) sec 150° (b) cot 135°

(a) $\sec 150° = \frac{1}{\cos 150°} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2\sqrt{3}}{3}$

(b) $\cot 135° = \frac{1}{\tan 135°} = \frac{1}{-1} = -1$

Answer: (a) $-\frac{2\sqrt{3}}{3}$ (b) -1

6 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 10 kg and length 2 m rests horizontally on supports at A and B. A load of mass 25 kg is placed on the beam 1 m from A. Take g = 9.8 m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 2 = 10 \times 9.8 \times 1 + 25 \times 9.8 \times 1$
$R_B \times 2 = 343$
Answer: 171.5 N

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7 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $x^2 + (y - 1)^2 = 169$ passes through P(12, −4).
Find the gradient of the tangent to the circle at P.

Centre C(0, 1).

Gradient of CP = $\frac{-4 - 1}{12 - 0} = -\frac{5}{12}$

The tangent is perpendicular to CP: gradient = $-1 \div (-\frac{5}{12})$

Answer: $\frac{12}{5}$

8 Integration by substitution (All boards)

Use the substitution $u = x^2 + 3$ to find
$\int 6x(x^2 + 3)^3 \, dx$.

$u = x^2 + 3$, so $du = 2x \, dx$

$\int 3u^3 \, du = \frac{3u^4}{4}$

Replace u: the answer below

Answer: $\frac{3(x^2 + 3)^4}{4} + c$

9 Linear and quadratic simultaneous equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve the simultaneous equations:
$y = x^2 - 2x + 8$, $y = 2x + 5$.

$x^2 - 2x + 8 = 2x + 5$
$x^2 - 4x + 3 = 0$
$(x - 3)(x - 1) = 0$
Use $y = 2x + 5$ to find each y.
Answer: x = 1, y = 7 and x = 3, y = 11

10 Area between a line and a curve (All boards)

The curve $y = 6x - x^2$ and the line $y = 3x$ meet at the origin and at P.
Find the area enclosed between them.

$6x - x^2 = 3x$ gives $3x - x^2 = 0$, so x = 0 or x = 3

Area = $\int_0^3 (3x - x^2) \, dx = [\frac{3x^2}{2} - \frac{x^3}{3}]$ from 0 to 3

$= \frac{3^3}{2} - \frac{3^3}{3} = \frac{9}{2}$

Answer: $\frac{9}{2}$