Day 260 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points A and B, 154 m apart, and cycle directly towards each other at constant speeds of 6 m s^{-1} and 8 m s^{-1}. Find the time until they meet.

Closing speed = 6 + 8 = 14 m s^{-1}

Time = $\frac{154}{14}$

Answer: 11 s

2 Integrate (ax + b)^n and related (All boards)

Find $\int \cos(4x + 1)\,dx$.

$\int \cos(ax + b)\,dx = \frac{1}{a}\sin(ax + b)$

Here a = 4

Answer: $\frac{1}{4}\sin(4x + 1) + c$

3 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−5, −5) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(x + 5) (b) y = f(−x)

(a) y = f(x + 5) subtracts 5 from x, so P maps to (−10, −5).
(b) y = f(−x) changes the sign of x, so P maps to (5, −5).
Answer: (a) (−10, −5) (b) (5, −5)

4 Combined mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The mean of 45 ages of the men is 80. The mean of 45 ages of the women is 69.
Find the mean of all 90 values.

Total of first group = 45 × 80 = 3600
Total of second group = 45 × 69 = 3105
Mean = 6705 ÷ 90
Answer: 74.5

5 Compound growth: find n (All boards)

£250 invested at 5% compound interest per year.
After how many whole years will it first exceed £750?

250 × 1.05^n > 750, so 1.05^n > 3

n > $\frac{\log 3}{\log 1.05}$ = 22.52

The least whole number of years is 23
Answer: 23 years

6 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(4x^4)^3 \div 2x^2$.

$(4x^4)^3 = 64x^{12}$

$64x^{12} \div 2x^2$: divide the numbers and subtract the powers

$= 32x^{10}$

Answer: $32x^{10}$

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7 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 65 kg rests on a smooth plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg\sin\alpha = 65 \times 9.8 \times \frac{5}{13}$

Perpendicular: $R = mg\cos\alpha = 65 \times 9.8 \times \frac{12}{13}$

Answer: P = 245, R = 588

8 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(60, 0.5). Find E(X) and Var(X).

E(X) = np = 60 × 0.5 = 30

Var(X) = npq = 60 × 0.5 × 0.5

Answer: E(X) = 30, Var(X) = 15

9 Maximum value from R form (All boards)

f(x) = 21 sin x + 20 cos x + 5.

Write f(x) in the form R sin(x + α) + 5, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{21^2 + 20^2} = 29$, $\tan\alpha = \frac{20}{21}$ so $\alpha = 43.6°$

f(x) = 29 sin(x + 43.6°) + 5

sin(x + α) has maximum value 1, so f(max) = 29 + 5

Answer: f(x) = 29 sin(x + 43.6°) + 5; maximum 34

10 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{14} (5r + 3)$.

This is an arithmetic series with first term 8 and common difference 5.

$S_{14} = \frac{14}{2}[2 \times 8 + 13 \times 5]$

= 567

Answer: 567