Day 257 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Degrees and radians (All boards)

(a) Convert $\frac{3\pi}{4}$ radians to degrees.

(b) Convert 15° to radians, giving your answer in terms of π.

(a) $\frac{3\pi}{4} \times \frac{180}{\pi} = 135°$

(b) $15 \times \frac{\pi}{180} = \frac{\pi}{12}$

Answer: (a) 135° (b) $\frac{\pi}{12}$ radians

2 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{7} (-2r - 4)$.

This is an arithmetic series with first term -6 and common difference -2.

$S_7 = \frac{7}{2}[2 \times (-6) + 6 \times (-2)]$

$= -84$

Answer: -84

3 Venn diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The Venn diagram shows probabilities for events A and B.
Find the value of x.

Probabilities sum to 1

$x = 1 - (0.15 + 0.15 + 0.1)$

Answer: 0.6

4 Equilibrium of forces given as vectors (AQA, Edexcel, OCR, MEI, Eduqas)

Three forces $(-6\mathbf{i} - 6\mathbf{j})$ N, $(p\mathbf{i} + q\mathbf{j})$ N and $(2\mathbf{i})$ N act on a particle which is in equilibrium. $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors. Find the values of p and q.

Sum of forces = 0:

i components: $p + (-6) + (2) = 0$

j components: $q + (-6) + (0) = 0$

Answer: $p = 4$, $q = 6$

5 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = \ln x + x - 4$.

Use the Newton-Raphson method with $x_0 = 3$ to find x_1,
giving your answer to 3 decimal places.

$f'(x) = \frac{1}{x} + 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 3 - \frac{0.09861}{1.333} = 2.9260$

Answer: $x_1 = 2.926$

6 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x - 4)^2 + (y - 2)^2 = 5$ passes through P(3, 4).
Find the gradient of the tangent to the circle at P.

Centre C(4, 2).

Gradient of CP = $\frac{4 - 2}{3 - 4} = -2$

The tangent is perpendicular to CP: gradient = $-1 \div (-2)$

Answer: $\frac{1}{2}$

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7 Maximum value from R form (All boards)

f(x) = 4 sin x + 3 cos x + 6.

Write f(x) in the form R sin(x + α) + 6, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{4^2 + 3^2} = 5$, $\tan \alpha = \frac{3}{4}$ so $\alpha = 36.9°$

f(x) = 5 sin(x + 36.9°) + 6

sin(x + α) has maximum value 1, so f(max) = 5 + 6

Answer: f(x) = 5 sin(x + 36.9°) + 6; maximum 11

8 Lift: tension in the cable (All boards)

A lift of mass 200 kg is raised by a vertical cable with constant acceleration 0.75 m s^{-2} upwards. Take g = 9.8 m s^{-2}. Find the tension in the cable.

Resolving upwards: $T - 200 \times 9.8 = 200 \times 0.75$

T = 1960 + 150

Answer: 2110 N

9 Normal approximation to the binomial (CIE)

X ~ B(60, 0.2). Use a normal approximation to find P(X ≥ 14), to 3 s.f.

np = 12 and npq = 9.6, so use Y ~ N(12, 9.6)

Continuity correction: P(Y > 13.5)

$z = (13.5 - 12) \div \sqrt{9.6}$

Answer: 0.314

10 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{6}{x - 2}$, $x \neq 2$.

Find $f^{-1}(x)$.

$y = \frac{6}{x - 2}$, so $y(x - 2) = 6$

$x - 2 = \frac{6}{y}$, so $x = \frac{6}{y} + 2$

Answer: $f^{-1}(x) = \frac{6}{x} + 2$