Day 256 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (4, −3) and B is (0, −7).
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint = $(\frac{4+0}{2}, \frac{-3+(-7)}{2}) = (2, -5)$

(b) $AB^2 = (-4)^2 + (-4)^2 = 16 + 16 = 32$

$AB = \sqrt{32} = 4\sqrt{2}$

Answer: (a) (2, −5) (b) $4\sqrt{2}$

2 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 12 cm and angle 1.25 radians.
Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 12 \times 1.25 = 15.0$ cm
(b) Perimeter = arc + 2r = 15.0 + 24 = 39.0 cm
Answer: (a) 15.0 cm (b) 39.0 cm

3 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 - 4x + 1$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 - 4x + 1$.

$x^2 - 4x = (x - 2)^2 - 4$

$x^2 - 4x + 1 = (x - 2)^2 - 3$

The minimum is where $(x - 2)^2 = 0$, i.e. $x = 2$

Answer: $(x - 2)^2 - 3$; minimum (2, −3)

4 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 3) and (4, 243).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 3$

$x = 4$: $3 \times b^4 = 243$, so $b^4 = 81$

$b = 3$

Answer: a = 3, b = 3

5 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(n, p) has mean 10 and variance 7.5.
Find n and p.

$np = 10$ and $npq = 7.5$

$q = 7.5 \div 10 = 0.75$, so $p = 0.25$

$n = 10 \div 0.25$

Answer: n = 40, p = 0.25

6 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arcsin(\frac{1}{2})$ (b) $\arctan(0)$

(a) $y = \arcsin(\frac{1}{2})$ means $\sin y = \frac{1}{2}$ with $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$: $y = \frac{\pi}{6}$

(b) $y = \arctan(0)$ means $\tan y = 0$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = 0$

Answer: (a) $\frac{\pi}{6}$ (b) 0

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7 Rough slope: limiting equilibrium or sliding (All boards)

A block is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{8}{15}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$
Parallel (limiting): $F = mg\sin\alpha$
$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\frac{8}{15}$

8 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(20, 0.8). Find E(X) and Var(X).

$E(X) = np = 20 \times 0.8 = 16$
$Var(X) = npq = 20 \times 0.8 \times 0.2$
Answer: E(X) = 16, Var(X) = 3.2

9 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(4\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(2\mathbf{i} + \mathbf{j})$ m at t = 0. Find the time when its position vector is $(14\mathbf{i} + 10\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$
$(14\mathbf{i} + 10\mathbf{j}) = (2\mathbf{i} + \mathbf{j}) + t(4\mathbf{i} + 3\mathbf{j})$
Equating i components: $14 = 2 + 4t$
Answer: 3 s

10 Geometric sequences (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric sequence has first term 2 and common ratio 3. Find the first term that exceeds 100000.

$u_n = 2 \times 3^{n-1} > 100000$
$u_{10} = 39366$ and $u_{11} = 118098$
Answer: $u_{11} = 118098$