Day 255 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 5 values: 2, 14, 17, 20, 11
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 64$, $\Sigma x^2 = 1010$, $n = 5$
Mean $= 64 \div 5 = 12.80$
Variance $= \frac{1010}{5} - (12.80)^2 = 38.16$
Standard deviation $= \sqrt{38.16}$
Answer: mean = 12.8, standard deviation = 6.18

2 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $(2x^3)^2 \div 4$.

$(2x^3)^2 = 4x^6$
$4x^6 \div 4$: divide the numbers and subtract the powers
$= x^6$
Answer: x^6

3 Resolving a force into components (All boards)

A rope pulls a box with a force of 24 N at an angle of 60° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal $= 24 \cos 60°$
Vertical $= 24 \sin 60°$
Answer: horizontal 12.0 N, vertical 20.8 N

4 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 13 and common ratio $\frac{1}{3}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.
$S_\infty = \frac{a}{1 - r} = \frac{13}{1 - \frac{1}{3}}$
$= \frac{39}{2}$
Answer: $\frac{39}{2}$

5 Area between a line and a curve (All boards)

The curve $y = 7x - x^2$ and the line $y = 1x$ meet at the origin and at P.
Find the area enclosed between them.

$7x - x^2 = 1x$ gives $6x - x^2 = 0$, so $x = 0$ or $x = 6$
Area $= \int_0^6 (6x - x^2)\,dx = [\frac{6x^2}{2} - \frac{x^3}{3}]$ from 0 to 6
$= \frac{6^3}{2} - \frac{6^3}{3} = 36$
Answer: 36

6 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x^2 + kx + 36 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 36 = 0$, so $k^2 = 144$
Answer: $k = \pm 12$

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7 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ax^n$. A graph of $\log_{10} y$ against $\log_{10} x$ is a straight line with gradient 3 and intercept 0.6. Find n and a, giving a to 3 significant figures.

$\log y = \log a + n \log x$
Gradient $n = 3$
$\log a = 0.6$ so $a = 10^{0.6} = 3.98$
Answer: n = 3, a = 3.98

8 Continuous random variable: probability and mean (CIE)

X has probability density function $f(x) = \frac{1}{9}x^2$ for $0 \leq x \leq 3$, and 0 otherwise.

Find $P(X < 2)$.

$P(X < 2) = \int$ from 0 to 2 of $\frac{1}{9}x^2\,dx$

$= [\frac{1}{27}x^3]$ from 0 to 2

Answer: $\frac{8}{27}$

9 Maximum value from R form (All boards)

$f(x) = 3 \sin x + 4 \cos x + 2$.
Write f(x) in the form $R \sin(x + \alpha) + 2$, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{3^2 + 4^2} = 5$, $\tan \alpha = \frac{4}{3}$ so $\alpha = 53.1°$
$f(x) = 5 \sin(x + 53.1°) + 2$
$\sin(x + \alpha)$ has maximum value 1, so f(max) = 5 + 2
Answer: f(x) = 5 sin(x + 53.1°) + 2; maximum 7

10 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is $(2t - 3)$ m s^{-2}.
When t = 0 its velocity is 3 m s^{-1}.
Find its velocity when t = 2.

$v = \int a\,dt = t^2 - 3t + c$
$t = 0$, $v = 3$ gives $c = 3$
When t = 2: $v = 4 + -6 + 3$
Answer: 1 m s^{-1}