Day 254 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 130 people, 66 are girls. 26 of the girls and 47 of the boys play an instrument.
One person is chosen at random.
Given that the person is a girl, find the probability that they play an instrument.

There are 66 girls
P = 26 ÷ 66
Answer: $\frac{13}{33}$

2 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (4, −6) and B is (14, −8).
Find (a) the midpoint of AB,
(b) the length of AB, as a surd in simplest form.

(a) Midpoint = $(\frac{4+14}{2}, \frac{-6+(-8)}{2}) = (9, -7)$
(b) $AB^2 = (10)^2 + (-2)^2 = 100 + 4 = 104$
$AB = \sqrt{104} = 2\sqrt{26}$
Answer: (a) (9, −7) (b) $2\sqrt{26}$

3 Factorised trig equation (All boards)

Solve (cos x − 1)(cos x + 1) = 0 for 0° ≤ x ≤ 360°.

Either cos x − 1 = 0 or cos x + 1 = 0
cos x = 1: x = 0°, 360°
cos x = −1: x = 180°
Answer: x = 0°, 180°, 360°

4 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is (4t + 2) m s^{-2}.
When t = 0 its velocity is 6 m s^{-1}.
Find its velocity when t = 4.

$v = \int a\,dt = 2t^2 + 2t + c$
t = 0, v = 6 gives c = 6
When t = 4: v = 32 + 8 + 6
Answer: 46 m s^{-1}

5 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is N(μ, 8^2). H_0: μ = 195, H_1: μ < 195.
A sample of 36 has mean $\bar{x}$ = 192.07.
Test at the 5% level. Critical value: z = −1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (192.07 - 195) \div (8 \div 6)$
z = −2.20
−2.20 is inside the critical region, so reject H_0
Answer: z = −2.20; reject H_0

6 Friction: acceleration along rough ground (All boards)

A block of mass 4 kg is pulled along rough horizontal ground by a horizontal force of 17.68 N.
The coefficient of friction is 0.4. Take g = 9.8 m s^{-2}. Find the acceleration.

R = 4 × 9.8 = 39.2
F = μR = 0.4 × 39.2 = 15.68
P − F = ma: 17.68 − 15.68 = 4 a
Answer: 0.5 m s^{-2}

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7 Exponential growth and decay model (All boards)

The number of bacteria is $N = 800\,e^{0.02t}$, where t is the time in hours.
Find (a) N when t = 10, (b) the time when N = 1600.
Give answers to 3 s.f.

(a) $N = 800\,e^{0.02 \times 10} = 800\,e^{0.2} = 977$

(b) $e^{0.02t} = 2$, so $0.02t = \ln 2$

$t = \dfrac{\ln 2}{0.02} = 34.7$ hours

Answer: (a) 977 (b) 34.7 hours

8 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\dfrac{5x - 3}{(x + 2)^2}$ in the form $\dfrac{A}{(x + 2)} + \dfrac{B}{(x + 2)^2}$.

$5x - 3 = A(x + 2) + B$
Coefficient of x: $A = 5$
Constant: $-3 = -10 + B$, so $B = -13$
Answer: A = 5, B = −13

9 Solve a^x = b using logarithms (All boards)

Solve $2 \times 7^x = 10$.
Give your answer to 3 significant figures.

Divide by 2: $7^x = 5$

$x = \dfrac{\log 5}{\log 7} = 0.827$

Answer: x = 0.827

10 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−6, −5) and B is (−4, −5). AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = (−5, −5)
$r^2 = (-4 - (-5))^2 + (-5 - (-5))^2 = 1$
Equation: $(x - a)^2 + (y - b)^2 = r^2$
Answer: $(x + 5)^2 + (y + 5)^2 = 1$