Day 251 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
= 0.8 × 0.9 + 0.2 × 0.9
Answer: 0.9

2 3D vectors: distance, magnitude and midpoint (All boards)

OA = $\begin{pmatrix} 2 \\ -3 \\ 3 \end{pmatrix}$ and OB = $\begin{pmatrix} 10 \\ -2 \\ 7 \end{pmatrix}$.

Find AB and |AB|, and a unit vector in the direction of AB.

AB = OB − OA = $\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$

|AB| = $\sqrt{8^2 + 1^2 + 4^2} = 9$

Unit vector = $\frac{1}{9}$ AB

Answer: AB = $\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$, |AB| = 9, unit vector $\frac{1}{9}\begin{pmatrix} 8 \\ 1 \\ 4 \end{pmatrix}$

3 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + 2x - 2$ is translated by the vector $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by (x − 4) and add −1:
y = f(x − 4) − 1
Expand and collect terms.
Answer: $y = x^2 - 6x + 5$

4 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\sin\theta = \frac{9}{41}$.

Find the exact values of cos θ and tan θ.

$\sin^2\theta + \cos^2\theta = 1$, so $\cos^2\theta = 1 - \frac{81}{1681} = \frac{1600}{1681}$

θ is acute, so $\cos\theta = \frac{40}{41}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{9}{40}$

Answer: $\cos\theta = \frac{40}{41}$, $\tan\theta = \frac{9}{40}$

5 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \sqrt{x + 3} - 2$.
State the largest possible domain and the range of f.

$x + 3 \geq 0$, so $x \geq -3$.
$\sqrt{\ldots} \geq 0$, so $f(x) \geq -2$.
Answer: Domain $x \geq -3$, range $f(x) \geq -2$

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6 Two particles over a smooth pulley (All boards)

Two particles of masses 1 kg and 3 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take g = 9.8 m s^{-2}. Find the acceleration and the tension in the string.

3 kg: $3 \times 9.8 - T = 3a$
1 kg: $T - 1 \times 9.8 = 1a$
Adding: $19.6 = 4a$, then $T = 1(9.8 + a)$

Answer: a = 4.9 m s^{-2}, T = 14.7 N

7 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 9 terms of the geometric series 768 + 384 + 192 + ...

$a = 768$, $r = \frac{1}{2}$, $n = 9$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{768(1 - (\frac{1}{2})^9)}{1 - \frac{1}{2}}$$

$= 1533$

Answer: 1533

8 2D vectors: magnitude and direction (All boards)

The vector $\mathbf{p} = 8\mathbf{i} - 15\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|\mathbf{p}| = \sqrt{8^2 + 15^2} = 17$

(b) Acute angle $= \tan^{-1}(\frac{15}{8}) = 61.9°$

p is in the fourth quadrant, so the angle is 298.1°

Answer: (a) 17 (b) 298.1°

9 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a cyclist.
(a) Find the acceleration in the first 6 s.
(b) Find the deceleration in the last 4 s.
(c) Find the total distance travelled.

(a) $a = \frac{11 - 5}{6}$

(b) $a = \frac{11 - 0}{4}$, a deceleration of 2.75

(c) area $= \frac{1}{2}(5 + 11) \times 6 + 11 \times 10 + \frac{1}{2} \times 4 \times 11$

Answer: (a) 1 m s^{-2} (b) 2.75 m s^{-2} (c) 180 m

10 Systematic sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A systematic sample of 15 students is taken from a numbered list of 165.
The first one selected is number 4. Find the number of the 9th one selected.

Interval $= 165 \div 15 = 11$
Number $= 4 + 8 \times 11$

Answer: 92