Day 249 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$x^2 + kx + 81 = 0$ has equal roots.
Find the possible values of k.

Equal roots: $b^2 - 4ac = 0$
$k^2 - 4 \times 1 \times 81 = 0$, so $k^2 = 324$
Answer: $k = \pm 18$

2 Driving force and resistance (All boards)

A car of mass 800 kg is driven along a straight horizontal road by a driving force of 600 N against a constant resistance of 200 N.
Find the acceleration of the car.

Resultant force = 600 − 200 = 400 N
F = ma: 400 = 800 a
Answer: 0.5 m s^{-2}

3 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(2x + 3)$ for $x = 1, 2, 3, 4, 5$.
(a) Find the value of k.
(b) Find $P(X \geq 3)$.

(a) Probabilities sum to 1: $k(5 + 7 + 9 + 11 + 13) = 1$
$k = \frac{1}{45}$

(b) $P(X \geq 3) = \frac{9}{45} + \frac{11}{45} + \frac{13}{45}$

Answer: (a) $k = \frac{1}{45}$ (b) $\frac{11}{15}$

4 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 12x + 4$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 12$
$dy/dx = 0$ gives $x = -2$ or $x = 2$
Substitute: $y = 20$ at $x = -2$ and $y = -12$ at $x = 2$
Answer: A (−2, 20), B (2, −12)

5 Car towing a caravan (All boards)

A car of mass 1000 kg tows a caravan of mass 500 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 200 N on the car, 150 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 150 = 1500 \times 0.5$
Caravan: $T - 150 = 500 \times 0.5$
Answer: D = 1100 N, T = 400 N

6 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 2x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{3}$, $y = 2x$

$= 1 + \frac{1}{3}(2x) + \frac{\frac{1}{3}((-\frac{2}{3}))}{2}(2x)^2$

$= 1 + \frac{2}{3}x - \frac{4}{9}x^2$

Valid when $|2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 + \frac{2}{3}x - \frac{4}{9}x^2$; $|x| < \frac{1}{2}$

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7 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $xy = 2$.

Find dy/dx in terms of x and y, and hence the gradient at (1, 2).

Differentiate both sides with respect to x (using dy/dx for y terms)

$\frac{dy}{dx} = -\frac{y}{x}$

At (1, 2): $\frac{dy}{dx} = -2$

Answer: $\frac{dy}{dx} = -\frac{y}{x}$; gradient -2

8 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+1} = \frac{1}{3} u_n + 8$.

The sequence converges to a limit L. Find L.

As $n \to \infty$, u_{n+1} and u_n both tend to L.

$L = \frac{1}{3} L + 8$

$L(1 - \frac{1}{3}) = 8$

Answer: L = 12

9 Normal approximation to the binomial (CIE)

X ~ B(60, 0.4). Use a normal approximation to find $P(X \geq 26)$, to 3 s.f.

np = 24 and npq = 14.4, so use Y ~ N(24, 14.4)

Continuity correction: P(Y > 25.5)

$z = (25.5 - 24) \div \sqrt{14.4}$

Answer: 0.346

10 Degrees and radians (All boards)

(a) Convert 60° to radians, giving your answer in terms of π.

(b) Convert $\frac{3\pi}{5}$ radians to degrees.

(a) $60° = 60 \times \frac{\pi}{180} = \frac{\pi}{3}$

(b) $\frac{3\pi}{5}$ radians $= \frac{3\pi}{5} \times \frac{180}{\pi} = 108°$

Answer: (a) $\frac{\pi}{3}$ radians (b) 108°