Day 247 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Iteration $x_{n+1} = g(x_n)$ (All boards)

$x_{n+1} = \frac{2}{x_n^2 + 1} + 1$, with $x_0 = 1$.

Find x_1, x_2 and x_3 to 3 decimal places.

$x_1 = 2.0000$

$x_2 = 1.4000$

$x_3 = 1.6757$

Answer: $x_1 = 2.000$, $x_2 = 1.400$, $x_3 = 1.676$

2 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 140 people, 90 are girls. 51 of the girls and 31 of the boys walk to work.
One person is chosen at random.
Given that the person walks to work, find the probability they are a girl.

82 people walk to work in total: 51 + 31

P = 51 ÷ 82

Answer: $\frac{51}{82}$

3 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{7} (4r + 6)$.

This is an arithmetic series with first term 10 and common difference 4.

$S_7 = \frac{7}{2}[2 \times 10 + 6 \times 4]$

$= 154$

Answer: 154

4 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 5 strips to estimate

$\int_0^2 e^{\frac{x^2}{2}}\,dx$,

giving your answer to 3 significant figures.

$h = \frac{2 - 0}{5} = 0.4$; x = 0, 0.4, 0.8, 1.2, 1.6, 2

y = 1.000, 1.083, 1.377, 2.054, 3.597, 7.389

Area $\approx \frac{h}{2}$[first + last + 2(sum of the rest)] $= \frac{0.4}{2}[1.000 + 7.389 + 2 \times 8.111]$

$= 4.92$

Answer: 4.92

5 Conditional probability formula (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A ∩ B) = 0.07 and P(B) = 0.7.
Find P(A | B).

P(A | B) = P(A ∩ B) ÷ P(B)

= 0.07 ÷ 0.7

Answer: 0.1

6 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 26 kg rests on a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{4}{3}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg \sin \alpha = 26 \times 9.8 \times \frac{4}{5}$

Perpendicular: $R = mg \cos \alpha = 26 \times 9.8 \times \frac{3}{5}$

Answer: P = 203.84, R = 152.88

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7 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 + 9 > 0$.

Solve $-x^2 + 9 = 0$: $-(x + 3)(x - 3) = 0$
Critical values $x = -3$ and $x = 3$.
The graph is an ∩-shaped parabola, so take the region between the roots.
Answer: $-3 < x < 3$

8 Integrate using partial fractions (All boards)

$\frac{5x + 9}{(x + 3)(x + 1)} \equiv \frac{A}{x + 3} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{5x + 9}{(x + 3)(x + 1)}\, dx$.

$5x + 9 \equiv A(x + 1) + B(x + 3)$
$x = -3$ gives $A = 3$; $x = -1$ gives $B = 2$
$\int \left(\frac{3}{x + 3} + \frac{2}{x + 1}\right) dx = 3 \ln|x + 3| + 2 \ln|x + 1| + c$
Answer: A = 3, B = 2; $3 \ln|x + 3| + 2 \ln|x + 1| + c$

9 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 1, −3, −7, −11, ...
Find (a) an expression for the nth term,
(b) the 17th term.

$a = 1$, $d = -4$
$u_n = a + (n - 1)d = 1 + (n - 1)(-4) = -4n + 5$
$u_{17} = -63$
Answer: (a) $-4n + 5$ (b) -63

10 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{4}{5}$. Find P and Q.

Horizontally: $P = 20\cos\alpha = 20 \times \frac{4}{5}$

Vertically: $Q = 20\sin\alpha = 20 \times \frac{3}{5}$

Answer: P = 16, Q = 12