Day 246 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Laws of logarithms: single log (All boards)

Write $2 \log a + 4 \log b - \log c$ as a single logarithm.

$2 \log a = \log a^2$ and $4 \log b = \log b^4$

$\log a^2 + \log b^4 - \log c = \log \frac{a^2b^4}{c}$

Answer: $\log \frac{a^2b^4}{c}$

2 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 3)^2 + (y + 3)^2 = 9$.
Determine whether the point P(4, 4) is inside, outside or on the circle.

Distance^2 from the centre $(3, -3)$ to P:
$(4 - 3)^2 + (4 - (-3))^2 = 1 + 49 = 50$
$r^2 = 9$, and $50 > 9$.
Answer: P is outside the circle

3 Compound growth: find n (All boards)

A population of 400 grows by 3% each year.
After how many whole years will it first exceed 1200?

$400 \times 1.03^n > 1200$, so $1.03^n > 3$
$n > \frac{\log 3}{\log 1.03} = 37.17$
The least whole number of years is 38
Answer: 38 years

4 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-2x^3 + 3x^2 - 2x + 6$ is divided by $(2x - 5)$.

Put $2x - 5 = 0$: $x = \frac{5}{2}$
Remainder = $f(\frac{5}{2}) = (-\frac{125}{4}) + \frac{75}{4} + (-5) + 6$
$= -\frac{23}{2}$
Answer: $-\frac{23}{2}$

5 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 4) and (3, 256).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 4$
$x = 3$: $4 \times b^3 = 256$, so $b^3 = 64$
$b = 4$
Answer: a = 4, b = 4

6 Interpreting the PMCC (AQA, Edexcel, OCR, MEI, Eduqas)

For data set A, $r = 0.54$. For data set B, $r = -0.77$.
Which data set has the stronger linear correlation? Give a reason.

Strength depends on the size of r, ignoring the sign.
$|0.54| = 0.54$ and $|-0.77| = 0.77$.
Answer: data set B, as |r| is larger

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7 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.

Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(-1, -3)$, so $y = a(x + 1)^2 - 3$.
The curve passes through $(0, -2)$:
$-2 = a(1)^2 - 3$, so $1 = 1\ a$
$a = 1$
Answer: $y = (x + 1)^2 - 3$

8 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(4\mathbf{i} - 4\mathbf{j})$ N and $(2\mathbf{i} + 6\mathbf{j})$ N act on a particle of mass 2 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant $= (4\mathbf{i} - 4\mathbf{j}) + (2\mathbf{i} + 6\mathbf{j})$
$= (6\mathbf{i} + 2\mathbf{j})$ N
$F = ma$: $a = \frac{1}{2}(6\mathbf{i} + 2\mathbf{j})$
Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

9 Normal approximation to the binomial (CIE)

$X \sim B(50, 0.6)$. Use a normal approximation to find $P(X \leq 33)$, to 3 s.f.

$np = 30$ and $npq = 12$, so use $Y \sim N(30, 12)$
Continuity correction: $P(Y < 33.5)$
$z = (33.5 - 30) \div \sqrt{12}$
Answer: 0.844

10 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moving in a straight line with constant acceleration increases its speed from 1 m s^{-1} to 15 m s^{-1} in 12 s. Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$
$s = \frac{(1 + 15)}{2} \times 12$
Answer: 96 m