Day 244 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Inverse functions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^2 + 5$, $x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

$y = x^2 + 5$, so $x^2 = y - 5$
$x \geq 0$, so $x = +\sqrt{y - 5}$
The domain of f^{-1} is the range of f: $y \geq 5$
Answer: $f^{-1}(x) = \sqrt{x - 5}$, $x \geq 5$

2 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 5) and (3, 135).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 5$
$x = 3$: $5 \times b^3 = 135$, so $b^3 = 27$
$b = 3$
Answer: $a = 5$, $b = 3$

3 Counting: arrangements and selections (CIE)

In how many ways can a team of 3 be chosen from 11 students?

$^{11}C_3 = \frac{11 \times 10 \times 9}{3 \times 2 \times 1}$
Answer: 165

4 The remainder theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the remainder when $-x^3 - 6x^2 + 5x - 6$ is divided by $(x + 3)$.

Remainder = $f(-3)$
$= 27 + (-54) + (-15) + (-6)$
$= -48$
Answer: -48

5 Stationary points of a cubic (All boards)

$y = x^3 + 3x^2 - 9x - 1$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 + 6x - 9 = 3(x + 3)(x - 1) = 0$
$x = -3$: $y = 26$; $x = 1$: $y = -6$
$d^2y/dx^2 = 6x + 6$: at $x = -3$ it is -12 (maximum), at $x = 1$ it is 12 (minimum)
Answer: $(-3, 26)$ maximum; $(1, -6)$ minimum

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity $(6\mathbf{i} + 19.6\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 19.6\,t - 4.9t^2$
$t = \frac{19.6}{4.9} = 4$
(b) Range $= 6 \times 4$
Answer: (a) 4 s (b) 24 m

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7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x + 1 < 3$ and $x^2 - 4 < 0$.

$2x + 1 < 3$ gives $x < 1$.
$x^2 - 4 < 0$: $(x + 2)(x - 2) < 0$, so $-2 < x < 2$.
Both are true where the two ranges overlap.
Answer: $-2 < x < 1$

8 Normal approximation to the binomial (CIE)

X ~ B(150, 0.3). Use a normal approximation to find P(X ≤ 48), to 3 s.f.

np = 45 and npq = 31.5, so use Y ~ N(45, 31.5)
Continuity correction: P(Y < 48.5)
$z = (48.5 - 45) \div \sqrt{31.5}$
Answer: 0.734

9 Solve a^x = b using logarithms (All boards)

Solve $6^{x+2} = 200$.
Give your answer to 3 significant figures.

Take logs: $(x + 2) \log 6 = \log 200$
$x + 2 = 2.957$, so $x = 0.957$
Answer: $x = 0.957$

10 Resolving a force into components (All boards)

A string pulls a box with a force of 40 N at an angle of 45° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 40 cos 45°
Vertical = 40 sin 45°
Answer: horizontal 28.3 N, vertical 28.3 N