Day 243 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Sam says that $n^2 + 3n + 27$ is prime for every positive integer n. Show that Sam is wrong.

Try n = 1, 2, 3, ... until the value is not prime.
When n = 3: 9 + 9 + 27 = 45
45 = 3 × 15, so it is not prime.
Answer: n = 3 gives 45 = 3 × 15

2 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:
(a) $4e^x + 3 \ln x$
(b) $e^{4x} + \sin 3x$

(a) $dy/dx = 4e^x + \frac{3}{x}$
(b) $dy/dx = 4e^{4x} + 3\cos 3x$
Answer: (a) $4e^x + \frac{3}{x}$ (b) $4e^{4x} + 3\cos 3x$

3 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.
Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is (−1, −1), so $y = a(x + 1)^2 - 1$.
The curve passes through (0, 0):
$0 = a(1)^2 - 1$, so $1 = 1\,a$
a = 1
Answer: $y = (x + 1)^2 - 1$

4 Power of an engine: driving force (CIE)

A train's engine works at a constant rate of 15 kW. Find the driving force when the vehicle is moving at 10 m s^{-1}.

P = Fv
15000 = F × 10
Answer: 1500 N

5 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The heights, X cm, of adult men are modelled by $N(60, 8^2)$.
Find P(52 < X < 56) to 3 s.f.

z-values: −1 and −0.5
P(−1 < Z < −0.5) from calculator
Answer: 0.150

6 Particle held by a string and a horizontal force (All boards)

A lamp of weight 3 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{4}{3}$. Find P and the tension T N.

Vertically: $T\cos\theta = 3$
Horizontally: $T\sin\theta = P$
Dividing: $P = 3\tan\theta$; $T = \sqrt{3^2 + P^2}$
Answer: P = 4, T = 5

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7 Maximum value from R form (All boards)

f(x) = 24 sin x + 7 cos x + 4.

Write f(x) in the form R sin(x + α) + 4, with α in degrees to 1 d.p., and state the maximum value of f(x).

$R = \sqrt{24^2 + 7^2} = 25$, $\tan \alpha = \frac{7}{24}$ so $\alpha = 16.3°$

f(x) = 25 sin(x + 16.3°) + 4

sin(x + α) has maximum value 1, so f(max) = 25 + 4

Answer: f(x) = 25 sin(x + 16.3°) + 4; maximum 29

8 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve |x + 1| = |x − 1|.

Squaring: $(x + 1)^2 = (x - 1)^2$

x + 1 = −(x − 1) (the case x + 1 = x − 1 has no solution)

2x = 0

Answer: x = 0

9 Parametric differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

x = 2 cos t, y = 3 sin t.

Find dy/dx in terms of t, and its value when $t = \frac{\pi}{4}$.

dx/dt = −2 sin t, dy/dt = 3 cos t

$\frac{dy}{dx} = \frac{3 \cos t}{-2 \sin t} = -\frac{3}{2} \cot t$

At $t = \frac{\pi}{4}$, cot t = 1, so $\frac{dy}{dx} = -\frac{3}{2}$

Answer: $\frac{dy}{dx} = -\frac{3}{2} \cot t$; $-\frac{3}{2}$

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 10^2)$. H_0: μ = 90, H_1: μ < 90.

A sample of 36 has mean $\bar{x} = 84.67$.

Test at the 5% level. Critical value: z = −1.645.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (84.67 - 90) \div (10 \div 6)$

z = −3.20

−3.20 is inside the critical region, so reject H_0

Answer: z = −3.20; reject H_0