Day 242 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length = $r\theta = 8 \times \frac{5\pi}{6} = \frac{20\pi}{3}$ cm

(b) Area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{5\pi}{6} = \frac{80\pi}{3}$ cm^2

Answer: (a) $\frac{20\pi}{3}$ cm (b) $\frac{80\pi}{3}$ cm^2

2 Comparing distributions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The heights in cm for two groups: Group A has mean 75, standard deviation 5; Group B has mean 70, standard deviation 19.
Compare the two sets of data.

The mean measures the average: Group A has the higher mean.
The standard deviation measures spread: Group A has the smaller standard deviation, so it is more consistent.
Answer: Group A has the higher mean; Group A is more consistent

3 Velocity at time t with constant acceleration (vectors) (AQA, Edexcel, OCR, MEI, Eduqas)

A particle has initial velocity $(6\mathbf{i} + 4\mathbf{j})$ m s^{-1} and constant acceleration $(-\mathbf{i} + 4\mathbf{j})$ m s^{-2}.
Find its velocity after 4 s.

$v = u + at$
$v = (6\mathbf{i} + 4\mathbf{j}) + 4(-\mathbf{i} + 4\mathbf{j})$
$= (6\mathbf{i} + 4\mathbf{j}) + (-4\mathbf{i} + 16\mathbf{j})$
Answer: $(2\mathbf{i} + 20\mathbf{j})$ m s^{-1}

4 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $|x - 2| < 2$.

$-2 < x - 2 < 2$
Add 2 to each part.
Answer: $0 < x < 4$

5 Binomial: finding parameters (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(n, p) has mean 6.4 and variance 3.84.
Find n and p.

$np = 6.4$ and $npq = 3.84$
$q = 3.84 \div 6.4 = 0.6$, so $p = 0.4$
$n = 6.4 \div 0.4$
Answer: $n = 16$, $p = 0.4$

6 Reciprocal graphs and asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = \frac{-6}{x + 3} - 4$.

Write down the equations of the asymptotes and the coordinates where the curve meets the axes.

Asymptotes: $x = -3$ and $y = -4$.

$y = 0$: $\frac{-6}{x + 3} = 4$, so $x = -\frac{9}{2}$

$x = 0$: $y = \frac{-6}{3} - 4 = -6$

Answer: $x = -3$, $y = -4$; $(-\frac{9}{2}, 0)$, $(0, -6)$

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7 Velocity-time graph: acceleration from two stages (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The graph shows the velocity of a skater.
(a) Find the acceleration in the first 4 s.
(b) Find the deceleration in the last 4 s.
(c) Find the total distance travelled.

(a) $a = \frac{12 - 6}{4}$

(b) $a = \frac{12 - 0}{4}$, a deceleration of 3

(c) area $= \frac{1}{2}(6 + 12) \times 4 + 12 \times 6 + \frac{1}{2} \times 4 \times 12$

Answer: (a) 1.5 m s^{-2} (b) 3 m s^{-2} (c) 132 m

8 Parallel vectors (All boards)

$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$.

Show that a and b are parallel and find the ratio |b| : |a|.

$\mathbf{b} = -2 \times \mathbf{a}$, since $-2 \times 2 = -4$ and $-2 \times 1 = -2$
So a and b are parallel and |b| : |a| = 2 : 1
Answer: b = −2 a, so parallel; ratio 2 : 1

9 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Rationalise the denominator of $\frac{3}{3 + \sqrt{7}}$.

Write your answer in the form $p + q\sqrt{7}$.

Multiply top and bottom by $3 - \sqrt{7}$:
$\frac{3(3 - \sqrt{7})}{(3)^2 - (\sqrt{7})^2} = \frac{3(3 - \sqrt{7})}{2}$
Answer: $\frac{9}{2} - \frac{3}{2}\sqrt{7}$

10 Integrate using partial fractions (All boards)

$\frac{-x - 7}{(x + 4)(x + 1)} \equiv \frac{A}{x + 4} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{-x - 7}{(x + 4)(x + 1)}\,dx$.

$-x - 7 \equiv A(x + 1) + B(x + 4)$
$x = -4$ gives $A = 1$; $x = -1$ gives $B = -2$
$\int \left(\frac{1}{x + 4} + \frac{-2}{x + 1}\right) dx = 1 \ln|x + 4| + -2 \ln|x + 1| + c$
Answer: A = 1, B = −2; ln |x + 4| − 2 ln |x + 1| + c