Day 241 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $5\,e^{4x}$

(b) $e^{5x} + \sin 4x$

(a) $dy/dx = 20\,e^{4x}$

(b) $dy/dx = 5\,e^{5x} + 4\cos 4x$

Answer: (a) $20\,e^{4x}$ (b) $5\,e^{5x} + 4\cos 4x$

2 Parametric equations: finding points (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

A curve has parametric equations

$x = t + 4$, $y = t^2 + 5$.

The point P has y-coordinate 14 and positive t.

Find the coordinates of P.

$t^2 + 5 = 14$, so $t^2 = 9$, giving $t = 3$ ($t > 0$)

$x = 1 \times 3 + 4 = 7$

Answer: (7, 14)

3 Integrate standard functions (All boards)

Find

(a) $\int 15\,e^{3x}\,dx$

(b) $\int 15\sec^2 3x\,dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) $= 5\,e^{3x} + c$

(b) $= 5\tan 3x + c$

Answer: (a) $5\,e^{3x} + c$ (b) $5\tan 3x + c$

4 Lift: tension in the cable (All boards)

A lift of mass 600 kg is raised by a vertical cable with constant acceleration 0.2 m s^{-2} upwards. Take g = 9.8 m s^{-2}.

Find the tension in the cable.

Resolving upwards: $T - 600 \times 9.8 = 600 \times 0.2$

$T = 5880 + 120$

Answer: 6000 N

5 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 130 people, 83 are women. 43 of the women and 29 of the men play an instrument.

One person is chosen at random.

Given that the person is a woman, find the probability that they play an instrument.

There are 83 women

$P = 43 \div 83$

Answer: $\frac{43}{83}$

6 Car towing a caravan (All boards)

A car of mass 800 kg tows a caravan of mass 400 kg on a level road with acceleration 0.25 m s^{-2}. Resistances: 400 N on the car, 200 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 400 - 200 = 1200 \times 0.25$

Caravan: $T - 200 = 400 \times 0.25$

Answer: D = 900 N, T = 300 N

Day 241 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.1$, H_1: $p > 0.1$.

In a sample, $x = 4$. It is given that $P(X \geq 4) = 0.133$.

Test H_0 at the 10% significance level.

Compare 0.133 with 0.1

0.133 > 0.1, so do not reject H_0

Answer: Do not reject H_0: insufficient evidence that p is greater than 0.1

8 Perpendicular bisector (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (6, 4) and B is (12, 6).

Find the equation of the perpendicular bisector of AB, in the form $ax + by = c$ with integer a, b and c.

Midpoint of AB = (9, 5)

Gradient of AB = $\frac{2}{6} = \frac{1}{3}$, so perpendicular gradient = -3

$y - 5 = -3(x - 9)$

Multiply out and rearrange.

Answer: 3x + y = 32

9 Partial fractions: find the constants (All boards)

Express $\frac{-5x - 21}{(x + 5)(x + 3)}$ in the form $\frac{A}{x + 5} + \frac{B}{x + 3}$.

Find the values of A and B.

$-5x - 21 \equiv A(x + 3) + B(x + 5)$

$x = -5$: $4 = A(-2)$, so $A = -2$

$x = -3$: $-6 = B(2)$, so $B = -3$

Answer: A = −2, B = −3

10 Partial fractions (Edexcel, OCR, MEI, CIE, AQA, Eduqas)

Express $\frac{5x + 6}{(x + 4)^2}$ in the form $\frac{A}{(x + 4)} + \frac{B}{(x + 4)^2}$.

$5x + 6 = A(x + 4) + B$

Coefficient of x: $A = 5$

Constant: $6 = -20 + B$, so $B = -14$

Answer: A = 5, B = −14