Day 240 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Recurrence relations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$, with
$u_1 = 3$ and $u_2 = 2$. Find u_6.

$u_3 = 2 + 3 = 5$
$u_4 = 5 + 2 = 7$, $u_5 = 7 + 5 = 12$
$u_6 = 12 + 7 = 19$
Answer: $u_6 = 19$

2 Integrate standard functions (All boards)

Find

(a) $\int \frac{4}{x}\,dx$

(b) $\int 8\sec^2 2x\,dx$

Use the standard integrals and divide by the coefficient of x where needed.
(a) = 4 ln |x| + c
(b) = 4 tan 2x + c
Answer: (a) 4 ln |x| + c (b) 4 tan 2x + c

3 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 10 to 30 has frequency 50 and is drawn with a bar of height 2.5 cm.
The class 30 to 35 has frequency 35. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 50 ÷ 20 = 2.5, so 2.5 cm represents 2.5
Second bar: 35 ÷ 5 = 7
Height = $2.5 \times \frac{7}{2.5}$
Answer: 7 cm

4 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 3 \geq 4$.

Solve $x^2 - 3x - 4 = 0$: $(x + 1)(x - 4) = 0$
Critical values $x = -1$ and $x = 4$.
The graph is a ∪-shaped parabola, so take the regions outside the roots.
Answer: $x \leq -1$ or $x \geq 4$

5 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 5 \ln x + x$ at the point where $x = 1$.

$dy/dx = 1 + 5/x$, so the gradient of the curve at $x = 1$ is 6
At $x = 1$, $y = 1$
$y - 1 = 6(x - 1)$
Answer: $y = 6x - 5$

6 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car starts from rest and accelerates uniformly at 2 m s^{-2} for 6 s, then travels at constant speed for a further 20 s. Find the total distance travelled.

Speed reached: $v = 2 \times 6 = 12$ m s^{-1}
Stage 1: $s = \frac{1}{2} \times 12 \times 6 = 36$
Stage 2: $s = 12 \times 20 = 240$
Answer: 276 m

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7 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = -\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (-1)(-3) + (1)(-3) + (-2)(1) = -2$

$|\mathbf{a}| = \sqrt{6}$, $|\mathbf{b}| = \sqrt{19}$

$\cos\theta = \frac{-2}{\sqrt{6} \times \sqrt{19}}$, so $\theta = 100.8°$

Answer: 100.8°

8 Car towing a caravan (All boards)

A car of mass 1200 kg tows a caravan of mass 400 kg on a level road with acceleration 1 m s^{-2}. Resistances: 200 N on the car, 150 N on the caravan. Find the driving force and the tow bar tension.

Whole system: $D - 200 - 150 = 1600 \times 1$

Caravan: $T - 150 = 400 \times 1$

Answer: D = 1950 N, T = 550 N

9 Straight line through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find an equation of the line through A(6, 5) and B(7, 7), in the form ax + by + c = 0, where a, b and c are integers.

Gradient $= \frac{7 - 5}{7 - 6} = 2$

$y - 5 = 2(x - 6)$

Multiply out and rearrange to integer coefficients.

Answer: $2x - y - 7 = 0$

10 Types of data (AQA, Edexcel, OCR, MEI, Eduqas)

Classify each variable as qualitative, discrete or continuous.
(a) the height of a plant
(b) the daily rainfall at a weather station
(c) the number of siblings a student has

(a) continuous: it is measured, so can take any value in a range
(b) continuous: it is measured, so can take any value in a range
(c) discrete: it is counted, so takes separate values

Answer: (a) continuous; (b) continuous; (c) discrete