Day 24 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Completing the square (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $x^2 - 4x + 6$ in the form $(x + p)^2 + q$, and state the coordinates of the minimum point of $y = x^2 - 4x + 6$.

$x^2 - 4x = (x - 2)^2 - 4$

$x^2 - 4x + 6 = (x - 2)^2 + 2$

The minimum is where $(x - 2)^2 = 0$, i.e. $x = 2$

Answer: $(x - 2)^2 + 2$; minimum $(2, 2)$

2 Binomial: P(X = k) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, 0.7)$. Find $P(X = 5)$, giving your answer to 3 s.f.

$P(X = 5) = {}^{10}C_5 \times 0.7^5 \times 0.3^5$

$= 252 \times 0.7^5 \times 0.3^5$

Answer: 0.103

3 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 3 strips to estimate

$\int_0^6 2^x \, dx$,

giving your answer to 3 significant figures.

$h = \frac{6 - 0}{3} = 2$; $x = 0, 2, 4, 6$

$y = 1.000, 4.000, 16.00, 64.00$

Area $\approx \frac{h}{2}$[first + last + 2(sum of the rest)] $= \frac{2}{2}[1.000 + 64.00 + 2 \times 20.00]$

$= 105$

Answer: 105

4 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(40, p)$. H_0: $p = 0.3$, H_1: $p > 0.3$.

Find the critical region for a test at the 1% significance level.

$P(X \geq 20) = 0.00625 \leq 0.01$

$P(X \geq 19) = 0.0148 > 0.01$

Answer: $X \geq 20$

5 Identity then solve (sin² to cos) (All boards)

Solve $-4\cos^2 x + 3 = 0$ for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $4\sin^2 x - 1 = 0$

Factorise and solve: $\sin x = \frac{1}{2}$ or $\sin x = -\frac{1}{2}$

$x = 30°, 150°, 210°, 330°$

Answer: $x = 30°, 150°, 210°, 330°$

6 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x - 2)^2 + (y - 1)^2 = 5$ passes through P(0, 2).

Find the gradient of the tangent to the circle at P.

Centre C(2, 1).

Gradient of CP $= \frac{2 - 1}{0 - 2} = -\frac{1}{2}$

The tangent is perpendicular to CP: gradient $= -1 \div (-\frac{1}{2})$

Answer: 2

7 Resolving a force into components (All boards)

A rope pulls a box with a force of 16 N at an angle of 60° above the horizontal. Find the horizontal and vertical components of the force, to 3 s.f.

Horizontal = 16 cos 60°

Vertical = 16 sin 60°

Answer: horizontal 8.00 N, vertical 13.9 N

8 Integrate using double angle formulae (All boards)

Use a double angle formula to find

$\int \cos^2 4x \, dx$.

$\cos 2\theta = 2\cos^2\theta - 1$, so $\cos^2 4x = \frac{1}{2}(1 + \cos 8x)$

Integrate term by term

Answer: $\frac{x}{2} + \frac{\sin 8x}{16} + c$

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -3x + k$ is a tangent to the curve

$y = x^2 - 7x - 1$.

Find the value of k.

$x^2 - 7x - 1 = -3x + k$

$x^2 - 4x + (-1 - k) = 0$

A tangent touches once: discriminant = 0

$(-4)^2 - 4(-1 - k) = 0$

Answer: $k = -5$

10 Driving force and resistance (All boards)

A car of mass 1200 kg is driven along a straight horizontal road by a driving force of 540 N against a constant resistance of 300 N.

Find the acceleration of the car.

Resultant force = 540 − 300 = 240 N

F = ma: 240 = 1200 a

Answer: 0.2 m s^{-2}