Day 239 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Mutually exclusive events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A and B are mutually exclusive, with P(A) = 0.55 and P(B) = 0.05.
(a) Find P(A ∪ B).
(b) Find the probability that neither A nor B happens.

(a) Mutually exclusive, so P(A ∩ B) = 0
P(A ∪ B) = 0.55 + 0.05 = 0.6
(b) 1 − 0.6 = 0.4
Answer: (a) 0.6 (b) 0.4

2 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{13} (2r + 3)$.

This is an arithmetic series with first term 5 and common difference 2.
$S_{13} = \frac{13}{2}[2 \times 5 + 12 \times 2]$
$= 221$
Answer: 221

3 Sector: arc length and area (diagram) (All boards)

The diagram shows sector OAB of a circle.
Find, in terms of π, (a) the arc length AB, (b) the area of the sector.

(a) Arc length $= r\theta = 8 \times \frac{\pi}{3} = \frac{8\pi}{3}$ cm
(b) Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 8^2 \times \frac{\pi}{3} = \frac{32\pi}{3}$ cm^2
Answer: (a) $\frac{8\pi}{3}$ cm (b) $\frac{32\pi}{3}$ cm^2

4 Car towing a caravan (All boards)

A car of mass 800 kg tows a caravan of mass 800 kg on a level road with acceleration 0.5 m s^{-2}. Resistances: 400 N on the car, 100 N on the caravan. Find the driving force and the tow bar tension.

Whole system: D − 400 − 100 = 1600 × 0.5
Caravan: T − 100 = 800 × 0.5
Answer: D = 1300 N, T = 500 N

5 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the satisfaction of the customers of a shop.
Give one advantage and one disadvantage of using quota sampling.

Advantage: no sampling frame is needed and each group is guaranteed to be represented.
Disadvantage: it is not random, so the interviewer may introduce bias.
Answer: Advantage: no sampling frame needed. Disadvantage: not random, interviewer bias

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A stone is projected from level ground with velocity (6**i** + 29.4**j**) m s^{-1} (**i** horizontal, **j** vertically up). Take g = 9.8 m s^{-2}.
(a) Find the time of flight.
(b) Find the horizontal range.

(a) Vertically: $0 = 29.4\,t - 4.9t^2$
$t = \frac{29.4}{4.9} = 6$
(b) Range = 6 × 6
Answer: (a) 6 s (b) 36 m

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7 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 - x)^8$.

$(1 + y)^8 = 1 + 8y + 28y^2 + 56y^3 + ...$ with $y = -x$

$= 1 + 8(-x) + 28(-x)^2 + 56(-x)^3$

Answer: $1 - 8x + 28x^2 - 56x^3$

8 Stationary points from a sketch (diagram) (All boards)

The diagram shows the curve $y = x^3 - 6x^2 + 9x + 4$.
A and B are turning points. Find the coordinates of A and B.

$dy/dx = 3x^2 - 12x + 9$

$dy/dx = 0$ gives $x = 1$ or $x = 3$

Substitute: $y = 8$ at $x = 1$ and $y = 4$ at $x = 3$

Answer: A (1, 8), B (3, 4)

9 Finding an unknown using the factor theorem (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$(x - 2)$ is a factor of $x^3 + kx^2 - 6x + 24$.
Find the value of k.

Let $f(x) = x^3 + kx^2 - 6x + 24$. Then $f(2) = 0$:

$8 + 4k + 12 = 0$

$4k = -20$

Answer: $k = -5$

10 Quotient rule (All boards)

Use the quotient rule to find the derivative of

$y = \frac{e^{4x}}{x}$.

Quotient rule: if $y = \frac{u}{v}$ then $dy/dx = \frac{u'v - uv'}{v^2}$

Differentiate the numerator and denominator, then substitute and simplify.

Answer: $dy/dx = \frac{e^{4x}(4x - 1)}{x^2}$