Day 238 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Connected rates of change (All boards)

A circular ripple has radius r cm, increasing at 0.1 cm/s.
Find the rate at which the area is increasing when r = 4, in terms of π.

$A = \pi r^2$, so $\frac{dA}{dr} = 2\pi r$

$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt} = 2\pi \times 4 \times 0.1$

Answer: 0.8π cm^2/s

2 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 8 m s^{-1} to 13 m s^{-1} as it travels 35 m. Find the acceleration.

$v^2 = u^2 + 2as$

$13^2 = 8^2 + 2 \times a \times 35$

$105 = 70\,a$

Answer: 1.5 m s^{-2}

3 Comparing distributions (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The journey times in minutes for two groups: Route A has mean 79, standard deviation 19; Route B has mean 61, standard deviation 17.
Compare the two sets of data.

The mean measures the average: Route A has the higher mean.
The standard deviation measures spread: Route B has the smaller standard deviation, so it is more consistent.

Answer: Route A has the higher mean; Route B is more consistent

4 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(3 - x)^6$.

Term in x^2: ${}^6C_2 \times 3^4 \times (-1\,x)^2$

$= 15 \times 81 \times 1$

Answer: 1215

5 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.
Use the small angle approximations to find an approximation for $1 - \cos 3\theta$.

Use $\cos 3\theta \approx 1 - \frac{3^2\theta^2}{2}$

So $1 - \cos 3\theta \approx \frac{9\,\theta^2}{2}$

Answer: $\frac{9\,\theta^2}{2}$

6 Moments: rod on a pivot with a particle (AQA, Edexcel, OCR, MEI)

A uniform rod AB of mass 15 kg and length 3 m is supported by a pivot 1.25 m from A. A particle of mass m kg is attached at A and the rod rests horizontally in equilibrium. Find m.

Moments about the pivot:

$m\,g \times 1.25 = 15\,g \times (1.5 - 1.25)$

$m = \frac{15 \times 0.25}{1.25}$

Answer: 3 kg

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7 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 3x)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = -\frac{1}{2}$, $y = 3x$

$= 1 + (-\frac{1}{2})(3x) + \frac{(-\frac{1}{2})((-\frac{3}{2}))}{2}(3x)^2$

$= 1 - \frac{3}{2}x + \frac{27}{8}x^2$

Valid when $|3x| < 1$, i.e. $|x| < \frac{1}{3}$

Answer: $1 - \frac{3}{2}x + \frac{27}{8}x^2$; $|x| < \frac{1}{3}$

8 Implicit differentiation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A curve has equation $x^2 + y^2 = 25$.
Find dy/dx in terms of x and y, and hence the gradient at $(-3, -4)$.

Differentiate both sides with respect to x (using dy/dx for y terms)

$dy/dx = -\frac{x}{y}$

At $(-3, -4)$: $dy/dx = -\frac{3}{4}$

Answer: $dy/dx = -\frac{x}{y}$; gradient $-\frac{3}{4}$

9 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.5$, H_1: $p < 0.5$.
In a sample, $x = 9$. It is given that $P(X \leq 9) = 0.1148$.
Test H_0 at the 10% significance level.

Compare 0.1148 with 0.1
0.1148 > 0.1, so do not reject H_0

Answer: Do not reject H_0: insufficient evidence that p is less than 0.5

10 Finding a quadratic from its turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a sketch of a quadratic curve.
Find its equation in the form $y = a(x - p)^2 + q$.

The turning point is $(-3, 2)$, so $y = a(x + 3)^2 + 2$.
The curve passes through $(0, -7)$:
$-7 = a(3)^2 + 2$, so $-9 = 9a$
$a = -1$

Answer: $y = -(x + 3)^2 + 2$