Day 236 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) 3 sin 3x

(b) e^{3x} + sin 3x

(a) dy/dx = 9 cos 3x

(b) dy/dx = 3 e^{3x} + 3 cos 3x

Answer: (a) 9 cos 3x (b) 3 e^{3x} + 3 cos 3x

2 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation $y = \frac{4}{3}x + 1$.

Find the equation of the line perpendicular to l that passes through $(-3, 1)$.

Gradient of l is $\frac{4}{3}$, so the perpendicular gradient is $-\frac{3}{4}$.

$y - 1 = -\frac{3}{4}(x - (-3))$

$y = -\frac{3}{4}x - \frac{5}{4}$

Answer: $y = -\frac{3}{4}x - \frac{5}{4}$

3 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity

$(4\mathbf{i} + 3\mathbf{j})$ m s^{-1}. It is at the point with position vector $(-\mathbf{i} + 4\mathbf{j})$ m at t = 0. Find the time when its position vector is $(11\mathbf{i} + 13\mathbf{j})$ m.

$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t$

$(11\mathbf{i} + 13\mathbf{j}) = (-\mathbf{i} + 4\mathbf{j}) + t(4\mathbf{i} + 3\mathbf{j})$

Equating i components: $11 = -1 + 4t$

Answer: 3 s

4 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = \frac{1}{x - 3} - 2$, $x \neq 3$.

Find the range of f.

$\frac{1}{x - 3}$ takes every real value except 0.

Adding -2: f(x) takes every value except -2.

Answer: f(x) takes all real values except -2

5 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: p = 0.3, H_1: $p \neq 0.3$. In a sample, x = 12.

Test at the 10% significance level.

Two-tailed test, so compare with $0.1 \div 2 = 0.05$

$P(X \geq 12) = 0.00514$

$0.00514 < 0.05$, so reject H_0

Answer: Reject H_0 ($P(X \geq 12) = 0.00514$)

6 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $-x^2 + 5x + 6 \geq 0$.

Solve $-x^2 + 5x + 6 = 0$: $-(x + 1)(x - 6) = 0$

Critical values $x = -1$ and $x = 6$.

The graph is an ∩-shaped parabola, so take the region between the roots.

Answer: $-1 \leq x \leq 6$

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7 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = **a**, OB = **b**, and P lies on AB with AP : PB = 3 : 2.
Find AB and AP in terms of **a** and **b**.

AB = OB − OA = **b** − **a**

AP = $\frac{3}{5}$ AB = $\frac{3}{5}$(**b** − **a**)

Answer: AB = b − a; AP = $\frac{3}{5}$(b − a)

8 Mean from a frequency table (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table summarises the number of goals scored in a game by 25 people.
Goals, x: 1, 2, 3, 4, 5
Frequency, f: 8, 3, 2, 3, 9
Find the mean.

Σfx = 8×1 + 3×2 + 2×3 + 3×4 + 9×5 = 77
Mean = Σfx ÷ Σf = 77 ÷ 25
Answer: 3.08

9 Two-stage motion: acceleration then constant speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist starts from rest and accelerates uniformly at 1.5 m s^{-2} for 10 s, then travels at constant speed for a further 10 s.
Find the total distance travelled.

Speed reached: v = 1.5 × 10 = 15 m s^{-1}

Stage 1: s = $\frac{1}{2}$ × 15 × 10 = 75

Stage 2: s = 15 × 10 = 150
Answer: 225 m

10 Separable differential equations (All boards)

Given that dy/dx = $2x^2$ and y = 0 when x = 0,
find y in terms of x.

y = ∫ $2x^2$ dx = $\frac{2x^3}{3}$ + c

x = 0, y = 0: c = 0

Answer: y = $\frac{2x^3}{3}$