Day 235 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Sampling methods (AQA, Edexcel, OCR, MEI, Eduqas)

A list of 270 customers is used. A random starting name in the first 5 is chosen, then every 5th name after it. Name the sampling method used.

This is systematic sampling: a regular interval is used after a random start.

Answer: systematic sampling

2 Solve a logarithmic equation (one log) (All boards)

Solve $\log_3 (2x + 1) = 2$.

$2x + 1 = 3^2 = 9$
$2x = 8$
$x = 4$

Answer: x = 4

3 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 15 kg rests on a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{4}{3}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg \sin \alpha = 15 \times 9.8 \times \frac{4}{5}$

Perpendicular: $R = mg \cos \alpha = 15 \times 9.8 \times \frac{3}{5}$

Answer: P = 117.6, R = 88.2

4 Definite integrals (exact) (All boards)

Find the exact value of

$\int_0^{\frac{\pi}{8}} 12 \cos 4x \, dx$.

[3 sin 4x] from 0 to $\frac{\pi}{8}$

$= 3 \sin \frac{\pi}{2} - 3 \sin 0 = 3$

Answer: 3

5 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(3\mathbf{i} + 3\mathbf{j})$ N and $(-\mathbf{i} - 7\mathbf{j})$ N act on a particle of mass 2 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant $= (3\mathbf{i} + 3\mathbf{j}) + (-\mathbf{i} - 7\mathbf{j})$
$= (2\mathbf{i} - 4\mathbf{j})$ N

F = ma: $a = \frac{1}{2}(2\mathbf{i} - 4\mathbf{j})$

Answer: $(\mathbf{i} - 2\mathbf{j})$ m s^{-2}

6 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 4x^2 - 9x + 36$
(a) Show that $(x - 3)$ is a factor of f(x).
(b) Factorise f(x) completely.

$f(3) = 27 + (-36) + (-27) + 36 = 0$, so $(x - 3)$ is a factor.

$f(x) = (x - 3)(x^2 - x - 12)$

$x^2 - x - 12 = (x + 3)(x - 4)$

Answer: (b) f(x) = (x + 3)(x − 3)(x − 4)

Day 235 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Integrate using partial fractions (All boards)

$\frac{x+1}{(x+3)(x+2)} \equiv \frac{A}{x+3} + \frac{B}{x+2}$.

Find A and B, and hence find $\int \frac{x+1}{(x+3)(x+2)}\,dx$.

$x + 1 \equiv A(x + 2) + B(x + 3)$

$x = -3$ gives $A = 2$; $x = -2$ gives $B = -1$

$\int \left(\frac{2}{x+3} + \frac{-1}{x+2}\right) dx = 2 \ln|x + 3| + -1 \ln|x + 2| + c$

Answer: $A = 2, B = -1$; $2 \ln|x + 3| - \ln|x + 2| + c$

8 Discriminant: equal roots (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The equation $x^2 + 2x + c = 0$ has equal roots.

Find the value of c.

Equal roots: $b^2 - 4ac = 0$

$2^2 - 4 \times 1 \times c = 0$

$4 = 4c$

Answer: $c = 1$

9 Grouped data: estimated mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.

Class: $10 \le x < 20$; $20 \le x < 30$; $30 \le x < 40$; $40 \le x < 50$

Frequency: 9, 6, 4, 2

Estimate the mean, to 3 s.f.

Midpoints: 15, 25, 35, 45

$\Sigma fx = 9\times15 + 6\times25 + 4\times35 + 2\times45 = 515$

Mean $\approx 515 \div 21$

Answer: 24.5

10 Equation of a transformed curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The curve $y = x^2 + x - 2$ is translated by the vector $\begin{pmatrix}4\\2\end{pmatrix}$.

Find the equation of the new curve, giving your answer in the form $y = ax^2 + bx + c$.

Replace x by $(x - 4)$ and add 2:

$y = f(x - 4) + 2$

Expand and collect terms.

Answer: $y = x^2 - 7x + 12$