Day 234 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arithmetic sequences: nth term (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic sequence begins 6, 4, 2, 0, ...
Find (a) an expression for the nth term,
(b) the 24th term.

$a = 6$, $d = -2$
$u_n = a + (n - 1)d = 6 + (n - 1)(-2) = -2n + 8$
$u_{24} = -40$
Answer: (a) $-2n + 8$ (b) -40

2 Natural logs and e^x (All boards)

Solve $e^{2x+1} = 3$.
Give your answer to 3 significant figures.

Take natural logs: $2x + 1 = \ln 3 = 1.099$
$x = 0.0493$
Answer: x = 0.0493

3 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of y = f(6x).

y = f(6x) is obtained from y = f(x) by a:

stretch parallel to the x-axis, scale factor $\frac{1}{6}$

Answer: stretch parallel to the x-axis, scale factor $\frac{1}{6}$

4 Binomial: cumulative probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(6, 0.2). Find P(X ≥ 1), giving your answer to 3 s.f.

$P(X \geq 1) = 1 - P(X \leq 0)$
$P(X \leq 0) = 0.2621$
Answer: 0.738

5 Area between a line and a curve (All boards)

The curve $y = 5x - x^2$ and the line $y = 3x$ meet at the origin and at P.
Find the area enclosed between them.

$5x - x^2 = 3x$ gives $2x - x^2 = 0$, so $x = 0$ or $x = 2$

Area $= \int_0^2 (2x - x^2)\,dx = \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 2

$= \frac{2^3}{2} - \frac{2^3}{3} = \frac{4}{3}$

Answer: $\frac{4}{3}$

6 Lift: tension in the cable (All boards)

A lift of mass 500 kg is raised by a vertical cable with constant acceleration 0.5 m s^{-2} upwards. Take g = 9.8 m s^{-2}.
Find the tension in the cable.

Resolving upwards: $T - 500 \times 9.8 = 500 \times 0.5$
$T = 4900 + 250$
Answer: 5150 N

7 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.3, P(B) = 0.2 and $P(A \cap B) = 0.01$.
(a) Find P(A | B).
(b) Are A and B independent? Give a reason.

(a) $P(A \mid B) = P(A \cap B) \div P(B) = 0.01 \div 0.2 = 0.05$
(b) $P(A) \times P(B) = 0.3 \times 0.2 = 0.06$
This does not equal $P(A \cap B) = 0.01$, so A and B are not independent
Answer: (a) 0.05 (b) not independent, as $P(A) \times P(B) \neq P(A \cap B)$

8 Particle on a smooth table connected to a hanging particle (All boards)

A 2 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 6 kg particle, as shown. The system is released from rest. Take g = 9.8 m s^{-2}. Find the tension.

Table particle: T = 2 a
Hanging particle: 6 × 9.8 − T = 6 a
Adding: 58.8 = 8 a
Answer: 14.7 N

9 Collinear points using vectors (All boards)

A(0, 2), B(12, −2) and C(21, −5).
Show that A, B and C are collinear and find AB : BC.

AB = $\begin{pmatrix} 12 \\ -4 \end{pmatrix}$ and BC = $\begin{pmatrix} 9 \\ -3 \end{pmatrix}$

AB = $4\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and BC = $3\begin{pmatrix} 3 \\ -1 \end{pmatrix}$, so AB and BC are parallel with common point B

Hence A, B, C are collinear and AB : BC = 4 : 3
Answer: collinear; AB : BC = 4 : 3

10 Transforming a point on a graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The point P(−4, −3) lies on the curve y = f(x).
Write down the coordinates of the image of P on
(a) y = f(−x) (b) y = 2f(x)

(a) y = f(−x) changes the sign of x, so P maps to (4, −3).
(b) y = 2f(x) multiplies y by 2, so P maps to (−4, −6).
Answer: (a) (4, −3) (b) (−4, −6)