Day 233 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Differentiate e^x, ln x, sin and cos (All boards)

Differentiate with respect to x:

(a) $3 \ln x$

(b) $e^{3x} + \sin 5x$

(a) $dy/dx = \frac{3}{x}$

(b) $dy/dx = 3e^{3x} + 5\cos 5x$

Answer: (a) $\frac{3}{x}$ (b) $3e^{3x} + 5\cos 5x$

2 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x - 2 \geq 0$.

Solve $x^2 - 2x = 0$: $(x + 0)(x - 2) = 0$

Critical values $x = 0$ and $x = 2$.

The graph is a ∪-shaped parabola, so take the regions outside the roots.

Answer: $x \leq 0$ or $x \geq 2$

3 Integrate standard functions (All boards)

Find

(a) $\int 8\cos 2x\, dx$

(b) $\int (4e^x + \frac{2}{x})\, dx$

Use the standard integrals and divide by the coefficient of x where needed.

(a) $= 4\sin 2x + c$

(b) $= 4e^x + 2\ln|x| + c$

Answer: (a) $4\sin 2x + c$ (b) $4e^x + 2\ln|x| + c$

4 Stratified sampling (AQA, Edexcel, OCR, MEI, Eduqas)

A stratified sample of size 25 is taken from 175 members: 28 Under 30, 77 30 to 50, 70 Over 50.

How many Over 50 should be in the sample?

Number $= 25 \times \frac{70}{175}$

$= 10$

Answer: 10

5 Exponential graph through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = ab^x$, where $b > 0$.

Find the values of a and b.

Through (0, 4): $a \times b^0 = 4$, so $a = 4$.

Through (2, 36): $4b^2 = 36$

$b^2 = 9$, so $b = 3$ $(b > 0)$

Answer: $a = 4$, $b = 3$

6 Catching up: constant speed against acceleration (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Car A passes a point O at constant speed 5 m s^{-1}.

At that instant car B starts from rest at O with constant acceleration 2 m s^{-2} in the same direction. Find the time when B catches up A.

Distances equal: $5t = \frac{1}{2} \times 2 \times t^2$

$t(5 - 1t) = 0$, so $t = 0$ or $t = 5 \div 1$

Answer: 5 s

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7 Equation of a reciprocal graph from asymptotes (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the graph of $y = \frac{k}{x - a} + c$ with its asymptotes. Find the values of a, c and k.

The asymptotes are $x = 1$ and $y = -1$, so $a = 1$ and $c = -1$.

Through (0, 1): $1 = \frac{k}{0 - 1} + -1$

$\frac{k}{-1} = 2$, so $k = -2$

Answer: $a = 1$, $c = -1$, $k = -2$

8 Vector geometry with a diagram (OAB) (All boards)

In the diagram, OA = $\mathbf{a}$, OB = $\mathbf{b}$, and P lies on AB with AP : PB = 1 : 2.
Find AB and OP in terms of $\mathbf{a}$ and $\mathbf{b}$.

$AB = OB - OA = \mathbf{b} - \mathbf{a}$

$AP = \frac{1}{3} AB = \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$OP = OA + AP = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

Answer: $\mathbf{AB} = \mathbf{b} - \mathbf{a}$; $\mathbf{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

9 Constant acceleration: distance from average speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry moving in a straight line with constant acceleration increases its speed from 3 m s^{-1} to 8 m s^{-1} in 12 s.
Find the distance it travels in this time.

$s = \frac{(u + v)}{2} t$

$s = \frac{(3 + 8)}{2} \times 12$

Answer: 66 m

10 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 8^2)$. H_0: $\mu = 160$, H_1: $\mu > 160$.

A sample of 9 has mean $\bar{x} = 165.07$.
Test at the 5% level. Critical value: $z = 1.645$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (165.07 - 160) \div (8 \div 3)$
$z = 1.90$
1.90 is inside the critical region, so reject H_0

Answer: $z = 1.90$; reject H_0