Day 232 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A is (−5, 0) and B is (5, 10). AB is a diameter of a circle.
Find the equation of the circle.

Centre = midpoint of AB = (0, 5)

$r^2 = (5 - 0)^2 + (10 - 5)^2 = 50$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $x^2 + (y - 5)^2 = 50$

2 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of $X \sim N(35, 2^2)$.
a and b are equal distances from the mean. The shaded area is 0.78.
Find P(X > b).

Unshaded area = 1 − 0.78 = 0.22

The two tails are equal by symmetry, so P(X > b) = 0.22 ÷ 2

Answer: 0.11

3 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 3 \ln x + x$ at the point where x = 1.

dy/dx = 1 + 3/x, so the gradient of the curve at x = 1 is 4

At x = 1, y = 1

y − 1 = 4(x − 1)

Answer: $y = 4x - 3$

4 Friction: constant speed, find the coefficient (All boards)

A crate of mass 15 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 88.2 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the crate and ground.

R = 15 × 9.8 = 147

Constant speed: F = P = 88.2

F = μR: 88.2 = μ × 147

Answer: 0.6

5 Rearrange and iterate (All boards)

(a) Show that $x^2 - 2x - 3 = 0$ can be rearranged to $x = \sqrt{2x + 3}$.
(b) Use x_{n+1} = this formula with $x_0 = 3$ to find x_1, x_2 to 3 d.p.

(a) $x^2 = 2x + 3$, so $x = \sqrt{2x + 3}$ (x > 0)

(b) $x_1 = 3.0000$, $x_2 = 3.0000$

Answer: (a) shown (b) $x_1 = 3.000$, $x_2 = 3.000$

6 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function f(x) = k(x + 2) for 0 ≤ x ≤ 4, and 0 otherwise.
Find the value of k.

∫ from 0 to 4 of k(x + 2) dx = 1

$k[\frac{x^2}{2} + 2x]$ from 0 to 4 = k(16) = 1

Answer: $k = \frac{1}{16}$

Day 232 **A Level Maths: C Tier 10-a-day: Model Solutions**

7 Separable differential equations (All boards)

Given that dy/dx = 3y and y = 5 when x = 0,
find y in terms of x.

$\frac{1}{y}$ dy = 3 dx, so ln y = 3x + c

x = 0, y = 5: c = ln 5

$y = e^{3x + \ln 5} = 5\,e^{3x}$

Answer: $y = 5\,e^{3x}$

8 Modulus equations (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve |x + 2| = |x − 2|.

Squaring: $(x + 2)^2 = (x - 2)^2$

x + 2 = −(x − 2) (the case x + 2 = x − 2 has no solution)

2x = 0

Answer: x = 0

9 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle P moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 3t^3 + 5t^2 + t + 2$.
Find its velocity when t = 1.

$v = \frac{ds}{dt} = 9t^2 + 10t + 1$

When t = 1: $v = 9 \times 1^2 + 10 \times 1 + 1$

Answer: 20 m s^{-1}

10 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x + 2 \geq 24$.

Solve $x^2 + 2x - 24 = 0$: (x + 6)(x − 4) = 0

Critical values x = −6 and x = 4.

The graph is a ∪-shaped parabola, so take the regions outside the roots.

Answer: $x \leq -6$ or $x \geq 4$