Day 231 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Modelling assumptions (All boards)

In an exam question, a string is described as light.
State what this assumption means for the motion or forces in the model.

Modelling assumption: light string.
The assumption means: i.ts mass is negligible, so the tension is the same throughout
Answer: Its mass is negligible, so the tension is the same throughout

2 Intercepts of a cubic graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = (x + 3)(x - 3)(x - 4)$. Write down the coordinates of the points where the curve meets the axes.

$y = 0$: $x = -3, 3, 4$
$x = 0$: $y = 3 \times (-3) \times (-4) = 36$
Answer: $(-3, 0)$, $(3, 0)$, $(4, 0)$, $(0, 36)$

3 Position vector under constant velocity (AQA, Edexcel, OCR, MEI, Eduqas)

A drone moves with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ m s^{-1}.
At time $t = 0$ its position vector is $(-3\mathbf{i} + 5\mathbf{j})$ m.
(a) Find its position vector when $t = 2$.
(b) Find the distance it travels in 2 s.

(a) $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}t = (-3\mathbf{i} + 5\mathbf{j}) + 2(3\mathbf{i} - 4\mathbf{j})$

(b) speed $= \sqrt{3^2 + 4^2} = 5$ m s^{-1}
distance $= 5 \times 2$
Answer: (a) $(3\mathbf{i} - 3\mathbf{j})$ m (b) 10 m

4 Arithmetic series: smallest n (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arithmetic series has first term 1 and common difference 6. Find the smallest n for which the sum of the first n terms exceeds 400.

$S_n = \frac{n}{2}[2 + 6(n - 1)] = \frac{n(6n - 4)}{2}$
$S_{11} = 341$ and $S_{12} = 408$
The sum first exceeds 400 when $n = 12$.
Answer: $n = 12$

5 Conditional probability: two-way tables (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Of 60 people, 21 are women. 10 of the women and 16 of the men own a bike.
One person is chosen at random.
Given that the person is a woman, find the probability that they own a bike.

There are 21 women
$P = 10 \div 21$
Answer: $\frac{10}{21}$

6 Laws of logarithms: single log (All boards)

Given that $\log_2 x = p$, write the following in terms of p:

(a) $\log_2 x^3$ (b) $\log_2 (4x)$

(a) $\log_2 x^3 = 3 \log_2 x = 3p$
(b) $\log_2 (4x) = \log_2 4 + \log_2 x = 2 + p$
Answer: (a) $3p$ (b) $p + 2$

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7 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of data on daily mean temperature and sunshine hours have r = 0.67.
Test, at the 5% significance level, for evidence of positive correlation. The critical value is 0.6215.

H_0: $\rho = 0$, H_1: $\rho > 0$
r = 0.67 is beyond the critical value 0.6215
Reject H_0
Answer: Reject H_0: there is evidence of positive correlation

8 Collinear points using vectors (All boards)

A(−1, −2), B(3, −10) and C(6, k).
Given that A, B and C are collinear, find k.

$\text{AB} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$, $\text{AC} = \begin{pmatrix} 7 \\ k + 2 \end{pmatrix}$

AC = $\frac{7}{4}$ AB, so $7 = \frac{7}{4} \times 4$, giving the multiple $\frac{7}{4}$

$k + 2 = \frac{7}{4} \times (-8)$, so $k = -16$

Answer: $k = -16$

9 Domain and range (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = (x + 1)^2 - 5$, $x \geq 0$.
Find the range of f.

For $x \geq 0$ the least value is at x = 0 (the vertex x = −1 is not allowed).
$f(0) = 1 - 5 = -4$, and f increases for x > 0.
Answer: $f(x) \geq -4$

10 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\cos\theta = \frac{7}{25}$.

Find the exact values of sin θ and tan θ.

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{49}{625} = \frac{576}{625}$

θ is acute, so $\sin\theta = \frac{24}{25}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{24}{7}$

Answer: $\sin\theta = \frac{24}{25}$, $\tan\theta = \frac{24}{7}$