Day 23 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{4x^2} \times 5x^{-2}$.

$\sqrt{4x^2} = (4x^2)^{\frac{1}{2}} = 2\,x^1$

$2\,x^1 \times 5x^{-2} = 10x^{-1}$

Answer: $10x^{-1}$

2 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use a small angle approximation to estimate cos 0.04, where 0.04 is in radians.

$$\cos\theta \approx 1 - \frac{\theta^2}{2}$$

$$\cos 0.04 \approx 1 - \frac{0.04^2}{2} = 0.9992$$

Answer: 0.9992

3 Sampling methods (AQA, Edexcel, OCR, MEI, Eduqas)

A reporter asks the first 10 people who walk out of a hospital for their views.
Name the sampling method used.

This is opportunity sampling: people are chosen only because they are available at the time.

Answer: opportunity sampling

4 Constant acceleration: $v^2 = u^2 + 2as$ (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with constant acceleration. Its speed increases from 4 m s^{-1} to 16 m s^{-1} as it travels 80 m. Find the acceleration.

$v^2 = u^2 + 2as$

$16^2 = 4^2 + 2 \times a \times 80$

$240 = 160\,a$

Answer: 1.5 m s^{-2}

5 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |x - 3|$. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where $x - 3 = 0$: $x = 3$

$x = 0$: $y = |-3| = 3$

Answer: Vertex (3, 0) ; y-axis (0, 3)

6 Differentiate tan, sec, cosec, cot (All boards)

Differentiate $y = \tan 3x$ with respect to x.

Use the standard result: $d/dx(\tan kx) = k\sec^2 kx$

Answer: $dy/dx = 3\sec^2 3x$

7 Grouped data: estimated median (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The table shows grouped data.
Class: $20 \le x < 30$; $30 \le x < 40$; $40 \le x < 50$; $50 \le x < 60$
Frequency: 4, 3, 3, 7
Estimate the median, to 3 s.f.

n = 17, so the median is the 8.5 th value, in class $40 \le x < 50$

Cumulative frequency before this class = 7

$$\text{Median} \approx 40 + \frac{(8.5 - 7)}{3} \times 10$$

Answer: 45.0

8 Identity then solve (sin² to cos) (All boards)

Solve $-2\sin^2 x - 3\cos x + 3 = 0$
for $0° \leq x \leq 360°$.

Use $\sin^2 x = 1 - \cos^2 x$: $2\cos^2 x - 3\cos x + 1 = 0$

Factorise and solve: $\cos x = 1$ or $\cos x = \frac{1}{2}$

$x = 0°, 60°, 300°, 360°$

Answer: x = 0°, 60°, 300°, 360°

9 Equilibrium of a particle under three forces (diagram) (All boards)

A particle is in equilibrium under the three forces shown. $\cos\alpha = \frac{4}{5}$. Find P and Q.

Horizontally: $P = 15\cos\alpha = 15 \times \frac{4}{5}$

Vertically: $Q = 15\sin\alpha = 15 \times \frac{3}{5}$

Answer: P = 12, Q = 9

10 Real roots and the discriminant (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of k for which
$2x^2 + 6x + k = 0$ has no real roots.

$b^2 - 4ac < 0$
$36 - 8k < 0$

Answer: $k > \frac{9}{2}$