Day 225 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) cot 315° (b) cosec 120°

(a) $\cot 315° = \frac{1}{\tan 315°} = \frac{1}{-1} = -1$

(b) $\operatorname{cosec} 120° = \frac{1}{\sin 120°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2\sqrt{3}}{3}$

Answer: (a) -1 (b) $\frac{2\sqrt{3}}{3}$

2 Constant acceleration: v = u + at (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A car moving in a straight line has initial speed 2 m s^{-1} and constant acceleration 1 m s^{-2}.
Find its speed after 5 s.

$v = u + at$
$v = 2 + 1 \times 5$
Answer: 7 m s^{-1}

3 Discrete random variable: finding k (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The discrete random variable X has $P(X = x) = k(x + 2)$ for $x = 1, 2, 3, 4, 5$.
(a) Find the value of k.
(b) Find $P(X \geq 3)$.

(a) Probabilities sum to 1: $k(3 + 4 + 5 + 6 + 7) = 1$

$k = \frac{1}{25}$

(b) $P(X \geq 3) = \frac{5}{25} + \frac{6}{25} + \frac{7}{25}$

Answer: (a) $k = \frac{1}{25}$ (b) $\frac{18}{25}$

4 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 2 \times 3^x - 54$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $3^x \to 0$, so $y \to -54$: asymptote $y = -54$.
$x = 0$: $y = 2 \times 3^0 - 54 = -52$
$y = 0$: $2 \times 3^x = 54$, so $3^x = 27$ and $x = 3$
Answer: $y = -54$; (0, −52), (3, 0)

5 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = k(x + 3)$ for $0 \leq x \leq 4$, and 0 otherwise.
Find the value of k.

$\int$ from 0 to 4 of $k(x + 3)\,dx = 1$

$k[\frac{x^2}{2} + 3x]$ from 0 to 4 $= k(20) = 1$

Answer: $k = \frac{1}{20}$

6 Transforming a graph with a labelled turning point (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x). The maximum point is shown. Write down the coordinates of the maximum point on the curve with equation
(a) $y = -f(x)$ (b) $y = 2f(x)$

(a) $y = -f(x)$: the point moves (it becomes a minimum) to (4, −4).
(b) $y = 2f(x)$: the maximum point moves to (4, 8).
Answer: (a) (4, −4) (b) (4, 8)

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7 2D vectors: magnitude and direction (All boards)

The vector $p = 7\mathbf{i} + 24\mathbf{j}$.
Find (a) |p|, (b) the angle p makes with the positive x-axis, measured anticlockwise (1 d.p.).

(a) $|p| = \sqrt{7^2 + 24^2} = 25$

(b) Acute angle = $\tan^{-1}(\frac{24}{7}) = 73.7°$

p is in the first quadrant, so the angle is 73.7°

Answer: (a) 25 (b) 73.7°

8 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \leq t \leq 4$) is $v = (-4t^2 + 16t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$a = \frac{dv}{dt} = 16 - 8t$

Maximum when $a = 0$: $t = \frac{16}{8} = 2$

$v = 16 \times 2 - 4 \times 2^2$

Answer: 16 m s^{-1} at t = 2

9 Find a and b in y = ab^x (All boards)

The curve $y = ab^x$ passes through (0, 3) and (3, 24).
Find the values of a and b.

$x = 0$: $y = a \times b^0 = a$, so $a = 3$

$x = 3$: $3 \times b^3 = 24$, so $b^3 = 8$

$b = 2$

Answer: a = 3, b = 2

10 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the coefficient of x^2 in the expansion of
$(1 + 3x)^6$.

Term in x^2: ${}^6C_2 \times 1^4 \times (3x)^2$

$= 15 \times 1 \times 9$

Answer: 135