Day 224 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Types of data (AQA, Edexcel, OCR, MEI, Eduqas)

Classify each variable as qualitative, discrete or continuous.
(a) a person's favourite colour
(b) the height of a plant
(c) the eye colour of a pupil

(a) qualitative: it is described by words, not numbers
(b) continuous: it is measured, so can take any value in a range
(c) qualitative: it is described by words, not numbers
Answer: (a) qualitative; (b) continuous; (c) qualitative

2 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of
(a) $\arctan(-1)$ (b) $\arccos(-1)$

(a) $y = \arctan(-1)$ means $\tan y = -1$ with $-\frac{\pi}{2} < y < \frac{\pi}{2}$: $y = -\frac{\pi}{4}$
(b) $y = \arccos(-1)$ means $\cos y = -1$ with $0 \leq y \leq \pi$: $y = \pi$
Answer: (a) $-\frac{\pi}{4}$ (b) π

3 Outliers (Edexcel, OCR, MEI)

For a data set $Q_1 = 10$ and $Q_3 = 24$.
A value is an outlier if it is more than 1.5 × IQR above Q_3 or below Q_1.
Determine whether -9 and -10 are outliers.

IQR = 24 − 10 = 14, so 1.5 × IQR = 21
Lower limit = 10 − 21 = −11; upper limit = 24 + 21 = 45
−9 is not an outlier; −10 is not an outlier
Answer: −9 is not an outlier; −10 is not an outlier

4 Integrate using partial fractions (All boards)

$\frac{-4x - 10}{(x + 4)(x + 1)} \equiv \frac{A}{x + 4} + \frac{B}{x + 1}$.

Find A and B, and hence find $\int \frac{-4x - 10}{(x + 4)(x + 1)}\,dx$.

$-4x - 10 \equiv A(x + 1) + B(x + 4)$
$x = -4$ gives $A = -2$; $x = -1$ gives $B = -2$
$\int \left(\frac{-2}{x + 4} + \frac{-2}{x + 1}\right) dx = -2 \ln|x + 4| + -2 \ln|x + 1| + c$
Answer: $A = -2$, $B = -2$; $-2 \ln|x + 4| - 2 \ln|x + 1| + c$

5 Modulus graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = |2x + 3|$. Write down the coordinates of the vertex and of the point where the graph meets the y-axis.

The vertex is on the x-axis where $2x + 3 = 0$: $x = -\frac{3}{2}$
$x = 0$: $y = |3| = 3$
Answer: Vertex $(-\frac{3}{2}, 0)$; y-axis $(0, 3)$

6 Moments: reactions on a uniform beam (AQA, Edexcel, OCR, MEI)

A uniform beam AB of mass 15 kg and length 10 m rests horizontally on supports at A and B. A load of mass 20 kg is placed on the beam 2.5 m from A. Take $g = 9.8$ m s^{-2}. Find the reaction at B.

Moments about A:
$R_B \times 10 = 15 \times 9.8 \times 5 + 20 \times 9.8 \times 2.5$
$R_B \times 10 = 1225$
Answer: 122.5 N

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7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both
$2x - 5 > -13$ and $x^2 + 8x + 12 < 0$.

$2x - 5 > -13$ gives $x > -4$.
$x^2 + 8x + 12 < 0$: $(x + 6)(x + 2) < 0$, so $-6 < x < -2$.
Both are true where the two ranges overlap.
Answer: $-4 < x < -2$

8 Power of an engine: driving force (CIE)

A train's engine works at a constant rate of 12 kW. Find the driving force when the vehicle is moving at 8 m s^{-1}.

$P = Fv$
$12000 = F \times 8$
Answer: 1500 N

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 2x + k$ is a tangent to the curve
$y = x^2 + 6x - 1$.
Find the value of k.

$x^2 + 6x - 1 = 2x + k$
$x^2 + 4x + (-1 - k) = 0$
A tangent touches once: discriminant = 0
$(4)^2 - 4(-1 - k) = 0$
Answer: $k = -5$

10 Integrate using double angle formulae (All boards)

Use a double angle formula to find
$\int \cos^2 2x \, dx$.

$\cos 2\theta = 2\cos^2\theta - 1$, so $\cos^2 2x = \frac{1}{2}(1 + \cos 4x)$
Integrate term by term
Answer: $\frac{x}{2} + \frac{\sin 4x}{8} + c$