Day 223 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Midpoint and length of a line segment (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

M(−3, 1) is the midpoint of the line segment AB.
A is the point (−1, 3). Find the coordinates of B.

x-coordinate: $\frac{-1 + x_B}{2} = -3$, so $x_B = -5$

y-coordinate: $\frac{3 + y_B}{2} = 1$, so $y_B = -1$

Answer: B(−5, −1)

2 Tension in a string supporting a hanging mass (All boards)

A particle of mass 2 kg hangs from a light string and is raised vertically with acceleration 1 m s^{-2}.
Taking g = 9.8 m s^{-2}, find the tension.

$T - mg = ma$
$T = 2 \times 9.8 + 2 \times 1$
Answer: 21.6 N

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Sam says that $n^2 + 3n + 13$ is prime for every positive integer n. Show that Sam is wrong.

Try n = 1, 2, 3, ... until the value is not prime.
When n = 9: 81 + 27 + 13 = 121
121 = 11 × 11, so it is not prime.
Answer: n = 9 gives 121 = 11 × 11

4 Solve a^x = b using logarithms (All boards)

Solve $2^x = 20$.
Give your answer to 3 significant figures.

Take logs of both sides: $x \log 2 = \log 20$
$x = \frac{\log 20}{\log 2} = 4.32$
Answer: x = 4.32

5 Normal distribution: diagram and symmetry (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the density of $X \sim N(40, 4^2)$.
a and b are equal distances from the mean. The shaded area is 0.74.
Find P(X > b).

Unshaded area = 1 − 0.74 = 0.26
The two tails are equal by symmetry, so P(X > b) = 0.26 ÷ 2
Answer: 0.13

6 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.

Use the small angle approximations to find an approximation for $\frac{\sin 4\theta}{\tan \theta}$.

$\sin 4\theta \approx 4\theta$, $\tan \theta \approx \theta$, so the ratio is about 4
So $\frac{\sin 4\theta}{\tan \theta} \approx 4$
Answer: 4

7 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: r = 5.
Centre (3, 5): $(x - a)^2 + (y - b)^2 = r^2$
Answer: $(x - 3)^2 + (y - 5)^2 = 25$

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8 Constant velocity vector: when does it reach a point? (AQA, Edexcel, OCR, MEI, Eduqas)

A particle moves with constant velocity
$(2\mathbf{i} + 5\mathbf{j})$ m s^{-1}. It is at the point with position vector $(3\mathbf{i} + 4\mathbf{j})$ m at t = 0. Find the time when its position vector is $(9\mathbf{i} + 19\mathbf{j})$ m.

$r = r_0 + vt$
$(9\mathbf{i} + 19\mathbf{j}) = (3\mathbf{i} + 4\mathbf{j}) + t(2\mathbf{i} + 5\mathbf{j})$
Equating i components: $9 = 3 + 2t$
Answer: 3 s

9 Solve a logarithmic equation (one log) (All boards)

Solve $\log_5 (5x + 4) = 2$.

$5x + 4 = 5^2 = 25$
$5x = 21$
$x = \frac{21}{5}$

Answer: $x = \frac{21}{5}$

10 Frequency density (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

In a histogram, the class 20 to 25 has frequency 40 and is drawn with a bar of height 4 cm. The class 25 to 35 has frequency 20. Find the height of its bar.

Frequency density = frequency ÷ class width
First bar: 40 ÷ 5 = 8, so 4 cm represents 8
Second bar: 20 ÷ 10 = 2
Height = $4 \times \frac{2}{8}$

Answer: 1 cm