Day 222 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with days of growth from 10 to 16, the regression line is H = 5 + 3d.
Estimate plant height (cm) when d = 25 and comment on the reliability.

H = 5 + 3 × 25 = 80
25 is outside the range 10 to 16, so this is extrapolation.
Answer: H = 80; estimate is unreliable, as 25 is outside the data range (extrapolation)

2 Solve a^x = b using logarithms (All boards)

Solve $5^{x+1} = 80$.
Give your answer to 3 significant figures.

Take logs: (x + 1) log 5 = log 80
x + 1 = 2.723, so x = 1.72
Answer: x = 1.72

3 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has common ratio $\frac{1}{3}$ and sum to infinity 27. Find the first term.

$S_\infty = \frac{a}{1-r}$, so $27 = \frac{a}{1-\frac{1}{3}}$

$a = 27 \times \frac{2}{3}$

Answer: a = 18

4 Particle held by a string and a horizontal force (All boards)

A ball of weight 12 N hangs from a light string, pulled aside by a horizontal force P N so that the string makes an angle θ with the vertical, where $\tan\theta = \frac{5}{12}$. Find P and the tension T N.

Vertically: $T\cos\theta = 12$
Horizontally: $T\sin\theta = P$
Dividing: $P = 12\tan\theta$; $T = \sqrt{12^2 + P^2}$
Answer: P = 5, T = 13

5 Exponential growth and decay model (All boards)

A radioactive sample has mass $M = 80\,e^{-0.02t}$ grams after t days.
Find the time for the mass to halve, to 3 significant figures.

$\frac{1}{2} = e^{-0.02t}$

$-0.02t = \ln\frac{1}{2} = -\ln 2$

$t = \frac{\ln 2}{0.02} = 34.7$ days

Answer: t = 34.7 days

6 Solving quadratics: surd answers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 + 10x - 4 = 0$, giving your answers in the form $p \pm \sqrt{q}$.

Complete the square: $(x + 5)^2 - 25 - 4 = 0$
$(x + 5)^2 = 29$
$x + 5 = \pm\sqrt{29}$
Answer: $x = -5 \pm \sqrt{29}$

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7 Rough slope: limiting equilibrium or sliding (All boards)

A block is on the point of sliding down a rough plane inclined at α to the horizontal, where $\tan\alpha = \frac{5}{12}$. Find the coefficient of friction.

Perpendicular: $R = mg\cos\alpha$
Parallel (limiting): $F = mg\sin\alpha$
$\mu = \frac{F}{R} = \tan\alpha$

Answer: $\frac{5}{12}$

8 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x + 4)^2 + (y + 1)^2 = 9$.
Determine whether the point P(8, −3) is inside, outside or on the circle.

Distance^2 from the centre (−4, −1) to P:
$(8 - (-4))^2 + (-3 - (-1))^2 = 144 + 4 = 148$
$r^2 = 9$, and $148 > 9$.
Answer: P is outside the circle

9 Conditional probability (Bayes) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The probability that a machine is set up wrongly is 0.2. If so, the probability that an item is faulty is 0.7; otherwise it is 0.4. Given that an item is faulty, find the probability that a machine is set up wrongly.

P(both) = 0.2 × 0.7 = 0.14; P(not first and second) = 0.8 × 0.4 = 0.32
P(second) = 0.14 + 0.32 = 0.46
P = 0.14 ÷ 0.46

Answer: $\frac{7}{23}$ **= 0.304**

10 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$.
Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (2)(3) + (-1)(4) + (4)(3) = 14$
$|\mathbf{a}| = \sqrt{21}$, $|\mathbf{b}| = \sqrt{34}$
$\cos\theta = \frac{14}{\sqrt{21} \times \sqrt{34}}$, so $\theta = 58.4°$

Answer: 58.4°