Day 221 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve a^x = b using logarithms (All boards)

Solve $1.5^{3x} = 10$.
Give your answer to 3 significant figures.

Take logs: $3x \log 1.5 = \log 10$

$x = \dfrac{\log 10}{3 \log 1.5} = 1.89$

Answer: x = 1.89

2 Exponential graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Sketch $y = 4 \times 2^x - 8$.
Write down the equation of the asymptote and the coordinates where the curve meets the axes.

As $x \to -\infty$, $2^x \to 0$, so $y \to -8$: asymptote $y = -8$.

$x = 0$: $y = 4 \times 2^0 - 8 = -4$

$y = 0$: $4 \times 2^x = 8$, so $2^x = 2$ and $x = 1$

Answer: y = −8 ; (0, −4), (1, 0)

3 Integrating velocity to find displacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line with velocity $v = (9t^2 - 2t + 6)$ m s^{-1} at time t seconds.
Find the distance travelled between t = 3 and t = 4 (the particle does not turn round).

$s = \int v \, dt = 3t^3 - t^2 + 6t$
Distance = $s(4) - s(3)$
$= 200 - 90$
Answer: 110 m

4 Solve a logarithmic equation (two logs) (All boards)

Solve $\ln(x + 12) - \ln x = \ln 5$.

$\ln \dfrac{x + 12}{x} = \ln 5$, so $\dfrac{x + 12}{x} = 5$

$x + 12 = 5x$, so $12 = 4x$

$x = 3$

Answer: x = 3

5 Hypothesis test for correlation (AQA, Edexcel, OCR, MEI, Eduqas)

8 pairs of data on height and arm span have r = 0.84.
Test, at the 5% significance level, for evidence of correlation. The critical value is ±0.7067.

$H_0: \rho = 0$, $H_1: \rho \neq 0$
r = 0.84 is beyond the critical value ±0.7067
Reject H_0
Answer: Reject H_0: there is evidence of correlation

6 Gradient and parallel or perpendicular lines (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line l has equation y = 2x + 3.
Find the equation of the line perpendicular to l that passes through (−2, −1).

Gradient of l is 2, so the perpendicular gradient is $-\frac{1}{2}$.

$y - (-1) = -\frac{1}{2}(x - (-2))$

$y = -\frac{1}{2}x - 2$

Answer: $y = -\frac{1}{2}x - 2$

7 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 7t + 10)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 7t + 10 = 0$
$(t - 2)(t - 5) = 0$

(b) $a = \frac{dv}{dt} = 2t - 7$; at $t = 2$, $a = 4 - 7$

Answer: (a) t = 2 and t = 5 (b) −3 m s^{-2}

8 Identity then solve (sin² to cos) (All boards)

Solve $-2\cos^2 x - \sin x + 2 = 0$
for $0° \leq x \leq 360°$.

Use $\cos^2 x = 1 - \sin^2 x$: $2\sin^2 x - \sin x = 0$

Factorise and solve: $\sin x = \frac{1}{2}$ or $\sin x = 0$

x = 0°, 30°, 150°, 180°, 360°

Answer: x = 0°, 30°, 150°, 180°, 360°

9 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of $y = \frac{1}{4}f(x)$.

$y = \frac{1}{4}f(x)$ is obtained from y = f(x) by a:

stretch parallel to the y-axis, scale factor $\frac{1}{4}$

Answer: stretch parallel to the y-axis, scale factor $\frac{1}{4}$

10 Selection without replacement (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A bag has 4 red and 6 blue marbles. Two are taken at random without replacement.
Find the probability that they are different colours.

$P(RB) = \frac{4}{10} \times \frac{6}{9}$, $P(BR) = \frac{6}{10} \times \frac{4}{9}$

Add: $\frac{4}{15} + \frac{4}{15}$

Answer: $\frac{8}{15}$