Day 220 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve a^x = b using logarithms (All boards)

Solve $2^{x+2} = 80$.
Give your answer to 3 significant figures.

Take logs: $(x + 2) \log 2 = \log 80$
$x + 2 = 6.322$, so $x = 4.32$
Answer: x = 4.32

2 Friction: constant speed, find the coefficient (All boards)

A crate of mass 2 kg is pulled along rough horizontal ground at constant speed by a horizontal force of 7.84 N. Take g = 9.8 m s^{-2}. Find the coefficient of friction between the crate and ground.

$R = 2 \times 9.8 = 19.6$
Constant speed: $F = P = 7.84$
$F = \mu R$: $7.84 = \mu \times 19.6$
Answer: 0.4

3 Laws of logarithms: single log (All boards)

Write $2 \log a + 3 \log b - \log c$ as a single logarithm.

$2 \log a = \log a^2$ and $3 \log b = \log b^3$

$\log a^2 + \log b^3 - \log c = \log \frac{a^2b^3}{c}$

Answer: $\log \frac{a^2b^3}{c}$

4 Rationalising the denominator (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\frac{15 + \sqrt{3}}{\sqrt{3}}$ in the form $p\sqrt{3} + q$.

$\frac{15 + \sqrt{3}}{\sqrt{3}} = \frac{15}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{3}}$
$= 5\sqrt{3} + 1$
Answer: $5\sqrt{3} + 1$

5 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with advertising spend (£s hundreds) from 4 to 14, the regression line is S = 11 + 6a.
Estimate sales (£s thousands) when a = 6 and comment on the reliability.

$S = 11 + 6 \times 6 = 47$
6 is inside the range 4 to 14, so this is interpolation.
Answer: S = 47; estimate is reliable, as 6 is within the data range (interpolation)

6 Two particles over a smooth pulley (All boards)

Two particles of masses 5 kg and 9 kg hang from the ends of a light inextensible string over a smooth pulley and are released from rest. Take g = 9.8 m s^{-2}. Find the acceleration and the tension in the string.

9 kg: $9 \times 9.8 - T = 9a$
5 kg: $T - 5 \times 9.8 = 5a$
Adding: $39.2 = 14a$, then $T = 5(9.8 + a)$
Answer: a = 2.8 m s^{-2}, T = 63 N

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7 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x - 1)^2 + (y - 2)^2 = 25$ passes through P(5, 5).
Find the gradient of the tangent to the circle at P.

Centre C(1, 2).

Gradient of CP = $\frac{5 - 2}{5 - 1} = \frac{3}{4}$

The tangent is perpendicular to CP: gradient = $-1 \div \frac{3}{4}$

Answer: $\mathbf{-\frac{4}{3}}$

8 Trapezium rule from a function (AQA, Edexcel, OCR, MEI, Eduqas)

Use the trapezium rule with 4 strips to estimate

$\int_0^4 \sqrt{x^3 + 2}\, dx$,

giving your answer to 3 significant figures.

h = $\frac{4 - 0}{4} = 1$; x = 0, 1, 2, 3, 4

y = 1.414, 1.732, 3.162, 5.385, 8.124

Area ≈ $\frac{h}{2}$[first + last + 2(sum of the rest)] = $\frac{1}{2}$[1.414 + 8.124 + 2 × 10.28]

= 15.0

Answer: 15.0

9 Finding a cubic from its graph (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve y = f(x), where f(x) is a cubic. Find f(x) in the form $k(x - a)(x - b)(x - c)$.

The curve crosses the x-axis at $x = -2$, 2 and 3.
$f(x) = k(x + 2)(x - 2)(x - 3)$
At (0, −12): $k \times 2 \times (-2) \times (-3) = -12$
$k = -1$
Answer: $\mathbf{f(x) = -(x + 2)(x - 2)(x - 3)}$

10 Binomial: mean and variance (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(100, 0.3). Find E(X) and Var(X).

$E(X) = np = 100 \times 0.3 = 30$
$Var(X) = npq = 100 \times 0.3 \times 0.7$
Answer: E(X) = 30, Var(X) = 21