Day 219 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Reciprocal trig functions: exact values (All boards)

Write down the exact values of
(a) sec 120° (b) sec 135°

(a) $\sec 120° = \frac{1}{\cos 120°} = \frac{1}{-\frac{1}{2}} = -2$

(b) $\sec 135° = \frac{1}{\cos 135°} = \frac{1}{-\frac{\sqrt{2}}{2}} = -\sqrt{2}$

Answer: (a) -2 (b) $-\sqrt{2}$

2 Standard deviation from raw data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Here are 5 values: 10, 12, 12, 15, 19
Find the mean and the standard deviation, to 3 s.f.

$\Sigma x = 68$, $\Sigma x^2 = 974$, $n = 5$
Mean $= 68 \div 5 = 13.60$
Variance $= \frac{974}{5} - (13.60)^2 = 9.840$
Standard deviation $= \sqrt{9.840}$
Answer: mean = 13.6, standard deviation = 3.14

3 Centre and radius of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $x^2 + y^2 + 8x - 65 = 0$.
Find the centre and the radius of the circle.

Complete the square in x and in y:
$(x + 4)^2 + y^2 = 81$
Centre $(-4, 0)$, radius $= 9$
Answer: Centre $(-4, 0)$, radius 9

4 Trapezium rule: strips shown (diagram) (AQA, Edexcel, OCR, MEI, Eduqas)

The diagram shows the curve $y = \sqrt{x + 2}$ from $x = 0$ to $x = 6$ divided into 3 strips of equal width.
Use the trapezium rule with these strips to estimate the shaded area to 3 s.f.

h = 2; y values: 1.414, 2.000, 2.449, 2.828
Area $\approx \frac{2}{2}[1.414 + 2.828 + 2(4.449)]$
$= 13.1$
Answer: 13.1

5 The factor theorem and factorising (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$f(x) = x^3 - 2x^2 - x + 2$
(a) Show that $(x + 1)$ is a factor of f(x).
(b) Factorise f(x) completely.

$f(-1) = (-1) + (-2) + 1 + 2 = 0$, so $(x + 1)$ is a factor.
$f(x) = (x + 1)(x^2 - 3x + 2)$
$x^2 - 3x + 2 = (x - 1)(x - 2)$
Answer: (b) $f(x) = (x + 1)(x - 1)(x - 2)$

6 Particle on a smooth table connected to a hanging particle (All boards)

A 5 kg particle on a smooth horizontal table is joined by a light string over a smooth pulley to a hanging 5 kg particle, as shown. The system is released from rest. Take $g = 9.8$ m s^{-2}. Find the tension.

Table particle: $T = 5a$
Hanging particle: $5 \times 9.8 - T = 5a$
Adding: $49 = 10a$
Answer: 24.5 N

7 Combining two inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the set of values of x satisfying both

$2x + 5 > -1$ and $x^2 - 36 < 0$.

$2x + 5 > -1$ gives $x > -3$.

$x^2 - 36 < 0$: $(x + 6)(x - 6) < 0$, so $-6 < x < 6$.

Both are true where the two ranges overlap.

Answer: $-3 < x < 6$

8 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-2\mathbf{i} + 7\mathbf{j})$ N and $(8\mathbf{i} - \mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant = $(-2\mathbf{i} + 7\mathbf{j}) + (8\mathbf{i} - \mathbf{j})$

$= (6\mathbf{i} + 6\mathbf{j})$ N

F = ma: $a = \frac{1}{3}(6\mathbf{i} + 6\mathbf{j})$

Answer: $(2\mathbf{i} + 2\mathbf{j})$ m s^{-2}

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(15, p)$. H_0: $p = 0.3$, H_1: $p > 0.3$.

Find the critical region for a test at the 10% significance level.

$P(X \geq 8) = 0.0500 \leq 0.1$

$P(X \geq 7) = 0.131 > 0.1$

Answer: $X \geq 8$

10 Integration by substitution (All boards)

Use $u = x^2 + 1$ to evaluate

$\int_0^1 2x(x^2 + 1)^2 \, dx$.

$u = x^2 + 1$, $du = 2x \, dx$; $x = 0$ gives $u = 1$, $x = 1$ gives $u = 2$

$\int_1^2 u^2 \, du = [\frac{u^3}{3}]$ from 1 to 2

$= \frac{2^3 - 1^3}{3}$

Answer: $\frac{7}{3}$