Day 214 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Quadratic inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $x^2 - 6x + 5 \leq 0$.

Solve $x^2 - 6x + 5 = 0$: $(x - 1)(x - 5) = 0$
Critical values $x = 1$ and $x = 5$.
The graph is a ∪-shaped parabola, so take the region between the roots.
Answer: $1 \leq x \leq 5$

2 Independent events (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

P(A) = 0.2, P(B) = 0.5 and P(A ∩ B) = 0.18.
(a) Find P(A | B).
(b) Are A and B independent? Give a reason.

(a) P(A | B) = P(A ∩ B) ÷ P(B) = 0.18 ÷ 0.5 = 0.36
(b) P(A) × P(B) = 0.2 × 0.5 = 0.1
This does not equal P(A ∩ B) = 0.18, so A and B are not independent
Answer: (a) 0.36 (b) not independent, as P(A) × P(B) ≠ P(A ∩ B)

3 Inverse trig functions (Edexcel, OCR)

Find the exact value, in radians, of

(a) $\arccos(-\frac{1}{2})$ (b) $\arccos(\frac{1}{2})$

(a) $y = \arccos(-\frac{1}{2})$ means $\cos y = -\frac{1}{2}$ with $0 \leq y \leq \pi$: $y = \frac{2\pi}{3}$

(b) $y = \arccos(\frac{1}{2})$ means $\cos y = \frac{1}{2}$ with $0 \leq y \leq \pi$: $y = \frac{\pi}{3}$

Answer: (a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{3}$

4 Gradient of a tangent to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The circle $(x - 2)^2 + y^2 = 13$ passes through P(4, 3).
Find the gradient of the tangent to the circle at P.

Centre C(2, 0).

Gradient of CP = $\frac{3 - 0}{4 - 2} = \frac{3}{2}$

The tangent is perpendicular to CP: gradient = $-1 \div \frac{3}{2}$

Answer: $-\frac{2}{3}$

5 Vertical motion under gravity: thrown upwards (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A stone is thrown vertically upwards with speed 39.2 m s^{-1}. Take g = 9.8 m s^{-2}.
(a) Find the time taken to reach its greatest height.
(b) Find the greatest height reached.

(a) $v = u + at$: $0 = 39.2 - 9.8t$, so $t = 4$
(b) $v^2 = u^2 + 2as$: $0 = 39.2^2 - 2 \times 9.8 \times s$
$s = \frac{1536.64}{19.6}$
Answer: (a) 4 s (b) 78.4 m

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6 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = x^3 - x - 6$.

Use the Newton-Raphson method with $x_0 = 2$ to find x_1 and x_2, giving your answer to 3 decimal places.

$f'(x) = 3x^2 - 1$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = 2 - \frac{0}{11.00} = 2.0000$

$x_2 = 2.0000 - \frac{0}{11.00} = 2.0000$

Answer: $x_1 = 2.000$, $x_2 = 2.000$

7 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(10, p)$. H_0: $p = 0.6$, H_1: $p < 0.6$.

Find the critical region for a test at the 1% significance level.

$P(X \leq 1) = 0.00168 \leq 0.01$

$P(X \leq 2) = 0.0123 > 0.01$

Answer: $X \leq 1$

8 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = 4x + k$ is a tangent to the curve $y = x^2 + 2x + 4$.

Find the value of k.

$x^2 + 2x + 4 = 4x + k$

$x^2 - 2x + (4 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(4 - k) = 0$

Answer: $k = 3$

9 Vector equation of a line (3D) (CIE)

A line has equation $\mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \\ -1 \end{pmatrix}$.

Show that the point $(-4, -8, 1)$ lies on the line.

Equate the first coordinates: $2 + 3t = -4$, so $t = -2$

Check the others: $-2 + 3 \times (-2) = -8$ and $-1 + (-1) \times (-2) = 1$

All three agree, so the point is on the line

Answer: Shown ($t = -2$)

10 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(-4\mathbf{i} - \mathbf{j})$ N and $(16\mathbf{i} + 5\mathbf{j})$ N act on a particle of mass 4 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant $= (-4\mathbf{i} - \mathbf{j}) + (16\mathbf{i} + 5\mathbf{j})$

$= (12\mathbf{i} + 4\mathbf{j})$ N

F = ma: $\mathbf{a} = \frac{1}{4}(12\mathbf{i} + 4\mathbf{j})$

Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}