Day 213 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Arc length and perimeter of a sector (All boards)

A sector of a circle has radius 9 cm and angle 1.5 radians.
Find (a) the arc length, (b) the perimeter of the sector.

(a) Arc length = $r\theta = 9 \times 1.5 = 13.5$ cm
(b) Perimeter = arc + 2r = 13.5 + 18 = 31.5 cm
Answer: (a) 13.5 cm (b) 31.5 cm

2 Moments: balancing a plank (AQA, Edexcel, OCR, MEI)

A light plank rests on a pivot. A child of mass 50 kg sits 1 m from the pivot on one side. Where must a child of mass 25 kg sit on the other side for the plank to balance horizontally?

Moments about the pivot:
$50\text{ g} \times 1 = 25\text{ g} \times d$
$d = \frac{50 \times 1}{25}$
Answer: 2 m from the pivot

3 Simplifying algebraic indices (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Simplify $\sqrt{16x^2} \times 2x^{-1}$.

$\sqrt{16x^2} = (16x^2)^{\frac{1}{2}} = 4\,x^1$
$4\,x^1 \times 2x^{-1} = 8$
Answer: 8

4 Newton-Raphson: one or two steps (AQA, Edexcel, OCR, MEI, Eduqas)

$f(x) = e^x - 5$.
Use the Newton-Raphson method with $x_0 = 1$ to find x_1 and x_2,
giving your answer to 3 decimal places.

$f'(x) = e^x$
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
$x_1 = 1 - \frac{-2.282}{2.718} = 1.8394$
$x_2 = 1.8394 - \frac{1.293}{6.293} = 1.6340$
Answer: $x_1 = 1.839$, $x_2 = 1.634$

5 Sigma notation (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Evaluate $\sum_{r=1}^{19} (5r + 1)$.

This is an arithmetic series with first term 6 and common difference 5.
$S_{19} = \frac{19}{2}[2 \times 6 + 18 \times 5]$
$= 969$
Answer: 969

6 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(25, p)$. H_0: $p = 0.5$, H_1: $p < 0.5$.
In a sample, $x = 8$. It is given that $P(X \leq 8) = 0.0539$.
Test H_0 at the 10% significance level.

Compare 0.0539 with 0.1
$0.0539 < 0.1$, so reject H_0
Answer: Reject H_0: evidence that p is less than 0.5

7 Angle between vectors (scalar product) (CIE)

$\mathbf{a} = 2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$.

Find the angle between a and b, to 1 decimal place.

$\mathbf{a}.\mathbf{b} = (2)(1) + (4)(3) + (-3)(3) = 5$

$|\mathbf{a}| = \sqrt{29}$, $|\mathbf{b}| = \sqrt{19}$

$\cos\theta = \frac{5}{\sqrt{29} \times \sqrt{19}}$, so $\theta = 77.7°$

Answer: 77.7°

8 Speed and direction from a velocity vector (AQA, Edexcel, OCR, MEI, Eduqas)

The velocity of a ball is $(-8\mathbf{i} + 15\mathbf{j})$ m s^{-1}, where $\mathbf{i}$ and $\mathbf{j}$ are perpendicular unit vectors.

(a) Find the speed.

(b) Find the acute angle between the direction of motion and the line of $\mathbf{i}$. Give 1 d.p.

(a) speed $= \sqrt{8^2 + 15^2} = \sqrt{289}$

(b) $\tan\theta = \frac{15}{8}$

$\theta = \tan^{-1}(\frac{15}{8})$

Answer: (a) 17 m s^{-1} (b) 61.9°

9 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that $12x + 9y = 37$.

Assume that integers x and y exist with $12x + 9y = 37$.
Then $3(4x + 3y) = 37$, so 37 would be a multiple of 3.
But $37 = 3 \times 12 + 1$, so it is not a multiple of 3.
This is a contradiction, so no such integers exist.

Answer: Proved

10 Continuous random variable: finding k (CIE)

The continuous random variable X has probability density function $f(x) = kx$ for $0 \leq x \leq 4$, and 0 otherwise.

Find the value of k.

$\int$ from 0 to 4 of $k\,x\,dx = 1$

$k \times \frac{4^2}{2} = 1$

Answer: $k = \frac{1}{8}$