Day 212 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Solve a logarithmic equation (one log) (All boards)

Solve $\log_2 x = 3$.

$x = 2^3$

Answer: $x = 8$

2 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(95, 8^2)$. Given that $P(X > a) = 0.8$, find a to 3 s.f.

$P(Z > z) = 0.8$, so $z = -0.8416$ (from the inverse normal function)

$a = 95 + -0.8416 \times 8$

Answer: 88.3

3 Constant acceleration: v = u + at (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A cyclist moving in a straight line has initial speed 1 m s^{-1} and constant acceleration 1.5 m s^{-2}. Find her speed after 7 s.

$v = u + at$

$v = 1 + 1.5 \times 7$

Answer: 11.5 m s^{-1}

4 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 8 terms of the geometric series 1 + 3 + 9 + ...

$a = 1$, $r = 3$, $n = 8$

$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{1(3^8 - 1)}{3 - 1}$$

$= 3280$

Answer: 3280

5 Estimate with a small angle (AQA, Edexcel, OCR, MEI, Eduqas)

Use $\sin\theta \approx \theta$ and $\cos\theta \approx 1 - \frac{\theta^2}{2}$ to estimate $\frac{\sin 0.15}{\cos 0.15}$.

$$\frac{\sin\theta}{\cos\theta} \approx \frac{\theta}{1 - \frac{\theta^2}{2}}$$

$$= \frac{0.15}{1 - 0.0112} = \frac{0.15}{0.9888} = 0.152$$

Answer: 0.152

6 Equation of a circle from a diagram (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a circle with the centre marked.
The circle touches the x-axis. Find the equation of the circle.

The circle touches the x-axis, so the radius equals the y-coordinate of the centre: $r = 4$.

Centre $(-1, 4)$: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x + 1)^2 + (y - 4)^2 = 16$

7 Maximum speed from calculus (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds ($0 \le t \le 4$) is $v = (-2t^2 + 8t)$ m s^{-1}. Find the maximum velocity and the time at which it occurs.

$$a = \frac{dv}{dt} = 8 - 4t$$

Maximum when $a = 0$: $t = \frac{8}{4} = 2$

$v = 8 \times 2 - 2 \times 2^2$

Answer: 8 m s^{-1} at $t = 2$

8 Separable differential equations (All boards)

Given that dy/dx = 2y and y = 2 when x = 0,
find y in terms of x.

$\frac{1}{y}$ dy = 2 dx, so ln y = 2x + c

x = 0, y = 2: c = ln 2

$y = e^{2x + \ln 2} = 2e^{2x}$

Answer: $y = 2e^{2x}$

9 Exponential graph through two points (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows the curve $y = ab^x$, where b > 0.
Find the values of a and b.

Through (0, 4): $a \times b^0 = 4$, so a = 4.

Through (2, 16): $4b^2 = 16$

$b^2 = 4$, so b = 2 (b > 0)

Answer: a = 4, b = 2

10 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, p). H_0: p = 0.4, H_1: p < 0.4.

Find the critical region for a test at the 5% significance level.

P(X ≤ 5) = 0.0294 ≤ 0.05

P(X ≤ 6) = 0.0736 > 0.05

Answer: X ≤ 5