Day 211 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Simplifying surds (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Write $\sqrt{75}$ in the form $p\sqrt{q}$, where q is an integer as small as possible.

$\sqrt{75} = \sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3}$

$= 5\sqrt{3}$

Answer: $5\sqrt{3}$

2 Solve trig equation (calculator, degrees) (All boards)

Solve $\sin x = 0.60$ for $0° \leq x \leq 360°$.

Give your answers to 1 decimal place.

Reference angle: $\sin^{-1}(0.600) = 36.9°$

sin is positive in the first and second quadrants.

$x = 36.9°$ or $x = 143.1°$

Answer: $x = 36.9°, 143.1°$

3 Normal distribution: probabilities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The times, X minutes, taken to complete a puzzle are modelled by $N(145, 3^2)$.

Find $P(X < 148)$ to 3 s.f.

Standardise: $z = (148 - 145) \div 3 = 1$

$P(Z < 1)$ from calculator

Answer: 0.841

4 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 + 2x)^5$.

$(1 + y)^5 = 1 + 5y + 10y^2 + 10y^3 + \ldots$ with $y = 2x$

$= 1 + 5(2x) + 10(2x)^2 + 10(2x)^3$

Answer: $1 + 10x + 40x^2 + 80x^3$

5 Integration by substitution (All boards)

Use the substitution $u = x^2 + 4$ to find

$\int \frac{3x}{x^2 + 4}\, dx$.

$u = x^2 + 4$, $du = 2x\, dx$, so $x\, dx = \frac{1}{2}\, du$

$\int \frac{3}{2u}\, du = \frac{3}{2} \ln |u|$

Answer: $\frac{3}{2} \ln (x^2 + 4) + c$

6 Projectile launched with velocity components (AQA, Edexcel, OCR, MEI)

A ball is projected from level ground with velocity $(3\mathbf{i} + 29.4\mathbf{j})$ m s^{-1} ($\mathbf{i}$ horizontal, $\mathbf{j}$ vertically up). Take g = 9.8 m s^{-2}.

(a) Find the time of flight.

(b) Find the greatest height reached.

(a) Vertically: $0 = 29.4\, t - 4.9t^2$

$t = \frac{29.4}{4.9} = 6$

(b) $0 = 29.4^2 - 2 \times 9.8 \times H$

$H = \frac{864.36}{19.6}$

Answer: (a) 6 s (b) 44.1 m

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7 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(10, p). H_0: p = 0.3, H_1: p ≠ 0.3. In a sample, x = 1.

Test at the 5% significance level.

Two-tailed test, so compare with 0.05 ÷ 2 = 0.025

P(X ≤ 1) = 0.149

0.149 > 0.025, so do not reject H_0

Answer: Do not reject H_0 (P(X ≤ 1) = 0.149)

8 Collinear points using vectors (All boards)

A(0, 0), B(12, −8) and C(21, −14).

Show that A, B and C are collinear and find AB : BC.

AB = $\begin{pmatrix} 12 \\ -8 \end{pmatrix}$ and BC = $\begin{pmatrix} 9 \\ -6 \end{pmatrix}$

AB = $4\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and BC = $3\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, so AB and BC are parallel with common point B

Hence A, B, C are collinear and AB : BC = 4 : 3

Answer: collinear; AB : BC = 4 : 3

9 Integrating acceleration to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its acceleration at time t seconds is (t − 5) m s^{-2}.

When t = 0 its velocity is 5 m s^{-1}.

Find its velocity when t = 4.

$v = \int a\,dt = 0.5t^2 - 5t + c$

t = 0, v = 5 gives c = 5

When t = 4: v = 8 + −20 + 5

Answer: -7 m s^{-1}

10 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 + 4x)^{-2}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with n = −2, y = 4x

$= 1 + (-2)(4x) + \frac{(-2)((-3))}{2}(4x)^2$

$= 1 - 8x + 48x^2$

Valid when |4x| < 1, i.e. $|x| < \frac{1}{4}$

Answer: $1 - 8x + 48x^2$; $|x| < \frac{1}{4}$