Day 210 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Regression: prediction and reliability (AQA, Edexcel, OCR, MEI, Eduqas)

For 7 data pairs, with advertising spend (£s hundreds) from 6 to 12, the regression line is S = 4 + 2a.
Estimate sales (£s thousands) when a = 11 and comment on the reliability.

S = 4 + 2 × 11 = 26

11 is inside the range 6 to 12, so this is interpolation.

Answer: S = 26; estimate is reliable, as 11 is within the data range (interpolation)

2 Constant acceleration: time to reach a speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A boat moving in a straight line increases its speed from 0 m s^{-1} to 17.5 m s^{-1} with constant acceleration 2.5 m s^{-2}. Find the time taken.

$v = u + at$

$17.5 = 0 + 2.5\,t$

Answer: 7 s

3 Proof by deduction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove that the sum of the squares of two integers that differ by 1 is always odd.

Let the integers be n and n + 1.

$n^2 + (n + 1)^2 = 2n^2 + 2n + 1$

$= 2(n^2 + 1n) + 1$

$2(n^2 + 1n)$ is even and 1 is odd, so the sum is odd.

Answer: Shown

4 3D vectors: distance, magnitude and midpoint (All boards)

$\mathrm{OA} = \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$ and $\mathrm{OB} = \begin{pmatrix} -1 \\ 10 \\ 8 \end{pmatrix}$.

Find AB and |AB|, and a unit vector in the direction of AB.

$\mathrm{AB} = \mathrm{OB} - \mathrm{OA} = \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$

$|\mathrm{AB}| = \sqrt{2^2 + 6^2 + 3^2} = 7$

Unit vector $= \frac{1}{7}\,\mathrm{AB}$

Answer: AB $= \begin{pmatrix} \mathbf{-2} \\ \mathbf{6} \\ \mathbf{3} \end{pmatrix}$**, |AB| = 7, unit vector** $\frac{1}{7}\begin{pmatrix} \mathbf{-2} \\ \mathbf{6} \\ \mathbf{3} \end{pmatrix}$

5 Binomial expansion: positive integer powers (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the first four terms, in ascending powers of x, of the expansion of $(1 - 2x)^4$.

$(1 + y)^4 = 1 + 4y + 6y^2 + 4y^3 + \ldots$ with $y = -2x$

$= 1 + 4(-2x) + 6(-2x)^2 + 4(-2x)^3$

Answer: $\mathbf{1 - 8x + 24x^2 - 32x^3}$

6 Find the angle of a sector (All boards)

A sector has radius 10 cm and area 37.5 cm^2.
Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 10^2 \times \theta = 37.5$, so $\theta = 0.75$

Arc length $= r\theta = 10 \times 0.75 = 7.50$ cm

Answer: $\boldsymbol{\theta}$ **= 0.75 radians; arc length 7.50 cm**

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7 Inclined plane: normal reaction and holding force (All boards)

A particle of mass 26 kg rests on a smooth plane inclined at α to the horizontal, where $\tan \alpha = \frac{12}{5}$.

It is held by a force P N up the slope. Take g = 9.8 m s^{-2}. Find P and the normal reaction.

Parallel: $P = mg \sin \alpha = 26 \times 9.8 \times \frac{12}{13}$

Perpendicular: $R = mg \cos \alpha = 26 \times 9.8 \times \frac{5}{13}$

Answer: P = 235.2, R = 98

8 Two-tailed binomial test (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(25, p). H_0: p = 0.4, H_1: p ≠ 0.4. In a sample, x = 13.

Test at the 5% significance level.

Two-tailed test, so compare with 0.05 ÷ 2 = 0.025

P(X ≥ 13) = 0.154

0.154 > 0.025, so do not reject H_0

Answer: Do not reject H_0 (P(X ≥ 13) = 0.154)

9 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line y = 3x + k is a tangent to the curve

$y = x^2 + x$.

Find the value of k.

$x^2 + x = 3x + k$

$x^2 - 2x + (0 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(0 - k) = 0$

Answer: k = −1

10 Separable differential equations (All boards)

Given that $dy/dx = 3x^2$ and y = 1 when x = 0,

find y in terms of x.

$y = \int 3x^2 \, dx = \frac{3x^3}{3} + c$

x = 0, y = 1: c = 1

Answer: $y = \frac{3x^3}{3} + 1$