Day 21 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Straight line from logarithmic graph (All boards)

Variables x and y satisfy $y = ab^x$. A graph of $\log_{10} y$ against x is a straight line with gradient 0.7 and intercept 0.9. Find a and b to 3 significant figures.

$\log y = \log a + x \log b$

$\log b = 0.7$ so $b = 10^{0.7} = 5.01$

$\log a = 0.9$ so $a = 10^{0.9} = 7.94$

Answer: a = 7.94, b = 5.01

2 Describing transformations of graphs (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Describe the single transformation that maps the graph of y = f(x) onto the graph of y = f(−x).

y = f(−x) is obtained from y = f(x) by a:
reflection in the y-axis

Answer: reflection in the y-axis

3 Sampling: advantages (AQA, Edexcel, OCR, MEI, Eduqas)

A researcher wants to find out the views of the students at a college.
Give one advantage and one disadvantage of using stratified sampling.

Advantage: each subgroup is represented in proportion to its size.
Disadvantage: the population must be divided into groups with known sizes, which takes effort.

Answer: Advantage: represents each group. Disadvantage: needs known group sizes

4 Area enclosed by a parabola (diagram) (All boards)

The diagram shows the curve $y = x(6 - x)$.
Find the area of the shaded region, between the curve and the x-axis.

The curve meets the x-axis where x = 0 and x = 6

Area $= \int_0^6 6x - x^2 \, dx = \left[\frac{6x^2}{2} - \frac{x^3}{3}\right]$ from 0 to 6

$= \frac{6^3}{2} - \frac{6^3}{3}$

$= 36$

Answer: 36

5 Geometric series: sum of n terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 5 terms of the geometric series 3 + 9 + 27 + ...

a = 3, r = 3, n = 5

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{3(3^5 - 1)}{3 - 1}$

$= 363$

Answer: 363

6 Resultant force as vectors and acceleration (AQA, Edexcel, OCR, MEI, Eduqas)

Two forces $(2\mathbf{i} + \mathbf{j})$ N and $(7\mathbf{i} + 2\mathbf{j})$ N act on a particle of mass 3 kg. No other horizontal forces act. Find the acceleration of the particle as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$.

Resultant $= (2\mathbf{i} + \mathbf{j}) + (7\mathbf{i} + 2\mathbf{j})$

$= (9\mathbf{i} + 3\mathbf{j})$ N

F = ma: $a = \frac{1}{3}(9\mathbf{i} + 3\mathbf{j})$

Answer: $(3\mathbf{i} + \mathbf{j})$ m s^{-2}

7 Normal distribution: inverse (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim N(95, 5^2)$. Given that P(X < a) = 0.6, find a to 3 s.f.

P(Z < z) = 0.6, so z = 0.2533 (from the inverse normal function)

$a = 95 + 0.2533 \times 5$

Answer: 96.3

8 Rough slope: limiting equilibrium or sliding (All boards)

A particle slides down a rough plane inclined at angle α to the horizontal, where $\sin\alpha = \frac{3}{5}$ and the coefficient of friction is 0.125. Take $g = 9.8$ m s^{-2}. Find the acceleration.

$R = mg\cos\alpha = m \times 9.8 \times \frac{4}{5}$

$mg\sin\alpha - \mu R = ma$

$a = 9.8 \times \frac{3}{5} - 0.125 \times 9.8 \times \frac{4}{5}$

Answer: 4.9 m s^{-2}

9 Geometric series: sum to infinity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A geometric series has first term 14 and common ratio $\frac{2}{5}$.

Find the sum to infinity.

$|r| < 1$, so the series converges.

$S_\infty = \frac{a}{1-r} = \frac{14}{1-\frac{2}{5}}$

$= \frac{70}{3}$

Answer: $\frac{70}{3}$

10 Optimisation (algebraic set-up) (All boards)

The product of x and $(27 - x^2)$ is $P = 27x - x^3$, $x > 0$.
Find the maximum value of P.

$dP/dx = 27 - 3x^2 = 0$, so $x^2 = 9$ and $x = 3$

$d^2P/dx^2 = -6x < 0$, so it is a maximum

$P = 27 \times 3 - 3^3 = 54$

Answer: P = 54