Day 209 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Linear inequalities (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Solve $8 < 3x - 4 \leq 17$.

Subtract -4 from each part: $12 < 3x \leq 21$
Divide each part by 3.
Answer: $4 < x \leq 7$

2 Driving force and resistance (All boards)

A car of mass 1000 kg is driven along a straight horizontal road by a driving force of 500 N against a constant resistance of 300 N.
Find the acceleration of the car.

Resultant force = $500 - 300 = 200$ N
F = ma: $200 = 1000\,a$
Answer: 0.2 m s^{-2}

3 Disproof by counter-example (AQA, Edexcel, OCR, MEI, Eduqas)

Sam says that $n^2 + 5n + 5$ is prime for every positive integer n. Show that Sam is wrong.

Try n = 1, 2, 3, ... until the value is not prime.
When n = 5: $25 + 25 + 5 = 55$
$55 = 5 \times 11$, so it is not prime.
Answer: n = 5 gives $55 = 5 \times 11$

4 Vertical motion: greatest height from speed (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

An arrow is projected vertically upwards with speed 35 m s^{-1} from ground level. Taking g = 9.8 m s^{-2}, find the maximum height it reaches above the ground.

At the top v = 0: $v^2 = u^2 + 2as$
$0 = 35^2 - 2 \times 9.8 \times s$
$s = \frac{1225}{19.6}$
Answer: 62.5 m

5 Arithmetic series: sum of terms (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Find the sum of the first 12 terms of the series
$-7 - 4 - 1 + 2 + \ldots$

$a = -7$, $d = 3$, $n = 12$
$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{12}{2}[2(-7) + 11 \times 3]$
$= 114$
Answer: 114

6 Solve trig equation (calculator, degrees) (All boards)

Solve $\cos x = -0.60$ for $0° \leq x \leq 360°$.
Give your answers to 1 decimal place.

Reference angle (positive value): $\cos^{-1}(0.600) = 53.1°$
cos is negative in the second and third quadrants.
$x = 126.9°$ or $x = 233.1°$
Answer: $x = 126.9°, 233.1°$

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7 Hypothesis test for a normal mean (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A population is $N(\mu, 5^2)$. H_0: $\mu = 45$, H_1: $\mu \neq 45$.

A sample of 64 has mean $\bar{x} = 46.69$.
Test at the 5% level. Critical value: $z = \pm 1.96$.

$z = (\bar{x} - \mu) \div (\sigma \div \sqrt{n}) = (46.69 - 45) \div (5 \div 8)$
$z = 2.70$
2.70 is inside the critical region, so reject H_0
Answer: $z = 2.70$; reject H_0

8 Differentiate tan, sec, cosec, cot (AQA, Edexcel, OCR, MEI, Eduqas)

Differentiate $y = \cot 2x$ with respect to x.

Use the standard result: $d/dx(\cot kx) = -k \operatorname{cosec}^2 kx$
Answer: $dy/dx = -2 \operatorname{cosec}^2 2x$

9 Normal approximation to the binomial (CIE)

$X \sim B(40, 0.4)$. Use a normal approximation to find $P(X \geq 13)$, to 3 s.f.

$np = 16$ and $npq = 9.6$, so use $Y \sim N(16, 9.6)$
Continuity correction: $P(Y > 12.5)$
$z = (12.5 - 16) \div \sqrt{9.6}$
Answer: 0.871

10 Stationary points of a cubic (All boards)

$y = x^3 - 3x^2 - 24x + 3$.
Find the coordinates of the stationary points and determine their nature.

$dy/dx = 3x^2 - 6x - 24 = 3(x + 2)(x - 4) = 0$
$x = -2$: $y = 31$; $x = 4$: $y = -77$
$d^2y/dx^2 = 6x - 6$: at $x = -2$ it is -18 (maximum), at $x = 4$ it is 18 (minimum)
Answer: $(-2, 31)$ maximum; $(4, -77)$ minimum