Day 208 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Line through two axis intercepts (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The diagram shows a line meeting the axes at A and B.
Find (a) the equation of AB in the form px + qy = r with integers p, q and r, (b) the length AB.

Gradient = $\frac{4-0}{0-(-1)} = 4$

Using y = mx + c with c = 4: y = 4x + 4, so 4x − y = −4

$AB^2 = 1^2 + 4^2 = 17$

Answer: (a) 4x − y = −4 (b) $\sqrt{17}$

2 Standard deviation from summary data (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

For 10 values, $\Sigma x = 390$ and $\Sigma x^2 = 15240$.
Find the mean and the standard deviation.

Mean = 390 ÷ 10 = 39

Variance = $\frac{15240}{10} - 39^2 = 1524 - 1521 = 3$

Standard deviation = $\sqrt{3}$

Answer: mean = 39, standard deviation = 1.73

3 Pythagorean identity: find cos and tan (All boards)

θ is acute and $\cos\theta = \frac{24}{25}$.

Find the exact values of sin θ and tan θ.

$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{576}{625} = \frac{49}{625}$

θ is acute, so $\sin\theta = \frac{7}{25}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{7}{24}$

Answer: $\sin\theta = \frac{7}{25}$, $\tan\theta = \frac{7}{24}$

4 Two objects moving towards each other (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

Two cyclists start at the same time from points P and Q, 35 m apart, and cycle directly towards each other at constant speeds of 3 m s^{-1} and 2 m s^{-1}. Find the time until they meet.

Closing speed = 3 + 2 = 5 m s^{-1}

Time = $\frac{35}{5}$

Answer: 7 s

5 Equation of a tangent or normal (All boards)

Find the equation of the tangent to the curve
$y = 2\sqrt{x} - x$ at the point where x = 1.

dy/dx = −1 + 1/sqrt(x), so the gradient of the curve at x = 1 is 0
At x = 1, y = 1
y − 1 = 0(x − 1)

Answer: y = 1

6 Tree diagrams (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The tree diagram shows probabilities for events R and L.
Find the probability that exactly one of R and L happens.

Exactly one: (R and L′) or (R′ and L)
= 0.4 × 0.3 + 0.6 × 0.2

Answer: 0.24

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7 Small angle approximations (AQA, Edexcel, OCR, MEI, Eduqas)

θ is small and measured in radians.

Use the small angle approximations to find an approximation for $\sin 4\theta \tan \theta$.

$\sin 4\theta \approx 4\theta$ and $\tan \theta \approx \theta$

So $\sin 4\theta \tan \theta \approx 4\theta^2$

Answer: $4\theta^2$

8 Binomial expansion: non-integer powers (Edexcel, AQA, OCR, MEI, Eduqas, CIE)

Expand $(1 - 2x)^{\frac{1}{3}}$ in ascending powers of x up to and including the term in x^2. State the range of values of x for which the expansion is valid.

$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \ldots$ with $n = \frac{1}{3}$, $y = -2x$

$= 1 + \frac{1}{3}(-2x) + \frac{\frac{1}{3}((-\frac{2}{3}))}{2}(-2x)^2$

$= 1 - \frac{2}{3}x - \frac{4}{9}x^2$

Valid when $|-2x| < 1$, i.e. $|x| < \frac{1}{2}$

Answer: $1 - \frac{2}{3}x - \frac{4}{9}x^2$; $|x| < \frac{1}{2}$

9 Braking: deceleration and time (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A lorry travelling at 20 m s^{-1} brakes uniformly and comes to rest after travelling 100 m.

(a) Find the deceleration.

(b) Find the time taken to stop.

(a) $v^2 = u^2 + 2as$: $0 = 20^2 + 2 \times a \times 100$

so $a = -2$, a deceleration of 2

(b) $s = \frac{(u + v)}{2}$ t: $100 = 10$ t

Answer: (a) 2 m s^{-2} (b) 10 s

10 Tangent condition for a line and a curve (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

The line $y = -4x + k$ is a tangent to the curve

$y = x^2 - 6x - 4$.

Find the value of k.

$x^2 - 6x - 4 = -4x + k$

$x^2 - 2x + (-4 - k) = 0$

A tangent touches once: discriminant = 0

$(-2)^2 - 4(-4 - k) = 0$

Answer: $k = -5$