Day 206 **A Level Maths: C Tier 10-a-day: Model Solutions**

1 Velocity zero (quadratic in t) (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its velocity at time t seconds is $v = (t^2 - 10t + 24)$ m s^{-1}.
(a) Find the times when the particle is at rest.
(b) Find its acceleration at the first of these times.

(a) $t^2 - 10t + 24 = 0$
$(t - 4)(t - 6) = 0$

(b) $a = \frac{dv}{dt} = 2t - 10$; at $t = 4$, $a = 8 - 10$

Answer: (a) $t = 4$ and $t = 6$ (b) -2 m s^{-2}

2 Equation of a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has centre (5, 5) and passes through the point (2, 4). Find the equation of the circle.

$r^2 = (2 - 5)^2 + (4 - 5)^2$
$= 9 + 1 = 10$

Equation: $(x - a)^2 + (y - b)^2 = r^2$

Answer: $(x - 5)^2 + (y - 5)^2 = 10$

3 Integrate (ax + b)^n and related (All boards)

Find $\int \sin(2x + 5)\,dx$.

$\int \sin(ax + b)\,dx = -\frac{1}{a}\cos(ax + b)$

Here $a = 2$

Answer: $-\frac{1}{2}\cos(2x + 5) + c$

4 Differentiating displacement to find velocity (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 - 3t^2 + 4t + 2$. Find its velocity when $t = 2$.

$v = \frac{ds}{dt} = 6t^2 - 6t + 4$

When $t = 2$: $v = 6 \times 2^2 + -6 \times 2 + 4$

Answer: 16 m s^{-1}

5 Partial fractions: find the constants (All boards)

Express $\frac{4x + 7}{(x + 2)(x + 1)}$ in the form $\frac{A}{x + 2} + \frac{B}{x + 1}$.
Find the values of A and B.

$4x + 7 \equiv A(x + 1) + B(x + 2)$
$x = -2$: $-1 = A(-1)$, so $A = 1$
$x = -1$: $3 = B(1)$, so $B = 3$

Answer: A = 1, B = 3

6 Binomial test using a given probability (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

$X \sim B(20, p)$. H_0: $p = 0.5$, H_1: $p > 0.5$.
In a sample, $x = 14$. It is given that $P(X \geq 14) = 0.0577$.
Test H_0 at the 5% significance level.

Compare 0.0577 with 0.05
0.0577 > 0.05, so do not reject H_0

Answer: Do not reject H_0: insufficient evidence that p is greater than 0.5

7 Find the angle of a sector (All boards)

A sector has radius 5 cm and area 6.25 cm^2.
Find the angle of the sector in radians and the arc length.

$\frac{1}{2} \times 5^2 \times \theta = 6.25$, so $\theta = 0.5$

Arc length = $r\theta = 5 \times 0.5 = 2.50$ cm

Answer: θ = 0.5 radians; arc length 2.50 cm

8 Position of a point relative to a circle (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

A circle has equation $(x - 4)^2 + (y + 2)^2 = 64$.
Determine whether the point P(−1, 0) is inside, outside or on the circle.

Distance^2 from the centre (4, −2) to P:

$(-1 - 4)^2 + (0 - (-2))^2 = 25 + 4 = 29$

$r^2 = 64$, and $29 < 64$.

Answer: P is inside the circle

9 Critical region (AQA, Edexcel, OCR, MEI, Eduqas, CIE)

X ~ B(15, p). H_0: p = 0.4, H_1: p > 0.4.
Find the critical region for a test at the 1% significance level.

$P(X \geq 11) = 0.00935 \leq 0.01$

$P(X \geq 10) = 0.0338 > 0.01$

Answer: X ≥ 11

10 Proof by contradiction (AQA, Edexcel, OCR, MEI, Eduqas)

Prove by contradiction that there are no integers x and y such that 20x + 15y = 81.

Assume that integers x and y exist with 20x + 15y = 81.
Then 5(4x + 3y) = 81, so 81 would be a multiple of 5.
But 81 = 5 × 16 + 1, so it is not a multiple of 5.
This is a contradiction, so no such integers exist.

Answer: Proved